circuit theory and networks - 01 (15th July)

* **charge :-** electric prop. atomic particles, which defines ēs deficiency/efficiency. (causes electrical force when placed in $\vec{E}/\vec{B}$.)

* **current :-** $i = \frac{dq}{dt}$ (rate of change of charge)

* **potential :-** workdone in bringing a +ve unit test charge from ∞ to a particular distance inside an $\vec{E}$. wrt.

* **potential difference :-** w^ done to moving a pt charge in a $\vec{E}$ from one pt to another.

* potential diff ≡ voltage $\left((v) = \frac{dw}{dq}\right)$ ↳ across the pt

Ammeter ← (current ↓ through some pt ↓ flow of ē) (voltmeter ↓ parallel)

↓ series.

* **EMF :-** Energy that creates pot. diff across two pts.

* **power :- (P) :** time rate of expanding/absorbing energy

$$P = \frac{dw}{dt}$$

$$= \frac{dw}{dq} \cdot \frac{dq}{dt}$$

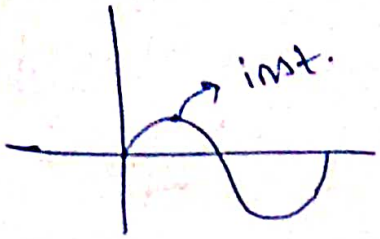
$$\boxed{P = v \cdot i}$$

(|| workdone (same dimension)

(lower case) v : Instaneous quantity → at a inst - ant

(upper case) V : avg quantity / RMS → AC

DC



$$P_{avg} = \int_0^T v_i dt$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

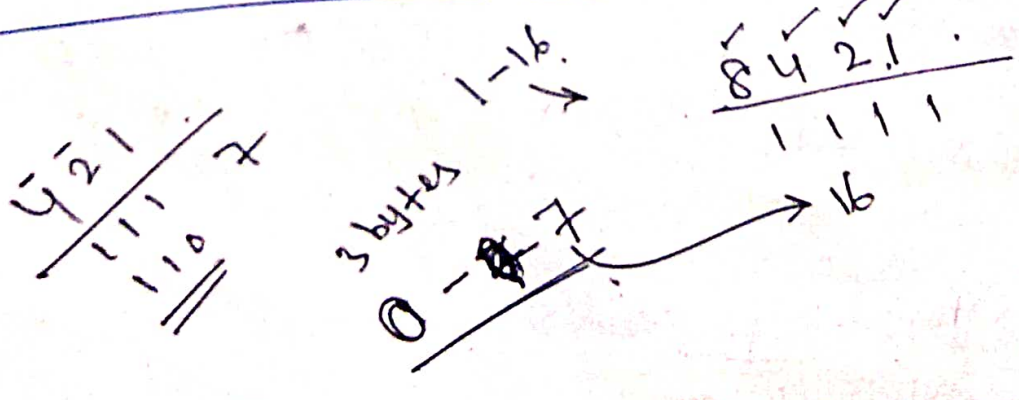
* Network | circuit :

↳ defined as inter connection of electrical devices for communicating or transforming electrical power from one pt to another pt.

⇒ ckt is that network which has atleast one closed path.

→ loop is a close path with no nodes pass through more than once.

→ mesh is a loop that does not contain any other loop within it.



Q) A total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC, calculate the current at $t = 0.5$ sec.

$$i = \frac{dq}{dt}$$

$$\frac{dq}{dt} = i = \frac{d}{dt}(5t \sin 4\pi t)$$

$$i = 5\{t \cdot 4\pi \cos 4\pi t + \sin 4\pi t\}$$

directly put $t = 0.5$

No Integration

$$= 5\left\{\frac{1}{2} \times 4\pi \cdot \cos 2\pi + \sin 2\pi\right\}$$

$$= 10\pi \text{ mA}$$

Q) Determine the total charge entering a terminal between $t = 1$s and $t = 2$s if the current passing the terminal is $i = (3t^2 - t)$ A

$$i = \frac{dq}{dt}$$

$$\int dq = \int_1^2 i\,dt = \int_1^2 3t^2 - t\,dt$$

$$q = [t^3]_1^2 - \left[\frac{t^2}{2}\right]_1^2$$

$$= (8 - 1) - [2 - 0.5]$$

$$= 4 - 1.5 = 5.5 \text{ C}$$

Q) Find the power delivered to an element at $t = 3$ ms, if the current entering the +ve terminal is $i = 5\cos 60\pi t$ A and the voltage is a) $V = 3i$ b) $V = 3\frac{di}{dt}$.

a) $p = vi$

$$= 3(5\cos 60\pi t)^2$$

$$= 25 \times 3(\cos 60\pi \times 3)^2$$

$$= 3 \times 25(75 \times 0.7123)$$

$$= 53.425 \text{ watt}$$

3 ms $= \frac{3}{1000}$ s

$\cos(60 \times \pi \times 3 \times 10^{-3})$ rad

$= 0.844$

$(0.844)^2 = 0.7123$

b) $p = 3 \times 5 \times 60\pi(-\sin 60\pi t) \times 5\cos 60\pi t$

$$= 5 \times 15 \times 30\pi \times (-\sin 120\pi t)$$

$$= 5 \times 450\pi \times (-\sin 120 \times \pi \times \frac{3}{1000})$$

$$= -65 \text{ watt}$$

$$= \pi \times 4500 \times (-\sin 12 \times \pi \times \frac{3}{100})$$

$$= 4500 \times (-0.028895)$$

$$= -130.0275 \text{ watt}.$$

$180° = \pi$ rad

$\sin(0.18\pi) \approx \sin(32.4°) \approx 0.529$

$\cos(0.18\pi) \approx \cos(32.4°) \approx 0.848$

$P = \frac{3}{-}\left[\frac{5}{-}(-\sin 60\pi t) \times 60\pi\right] \times 5\cos 60\pi t$

$= 4500 \times \left(-2 \sin 120\pi \times \frac{3}{1000}\right)$

$= 9000 \times \left(-\sin\left(120\pi \times \frac{3}{1000}\right)\right)$

* $\pi = 180$ (when calculator is in degree mode)
  $= 3.14$ (" radian)
* $0.596 \approx 0.6$ ~~rad~~ calculator (degree)

$\Rightarrow P = 3\left[5(-\sin 60\pi t) \times 60\pi\right] 5\cos 60\pi t$

$(3\frac{di}{dt}) \times i = P$

////

$\Rightarrow t = \frac{3}{1000}$ s

$\Rightarrow v\big|_{3ms} = 3\frac{di}{dt}\Big|_{3ms}$

$= 3(-300\pi)\sin(60\pi(0.003))$

$= -900\pi \sin(0.18\pi)$

$i\big|_{3ms} = 5\cos(60\pi(0.003))$

$\frac{d}{dt} i = \frac{d}{dt}\left(\frac{5}{-}\cos\frac{60\pi t}{-}\right)$

$= -300\pi \sin 60\pi t$

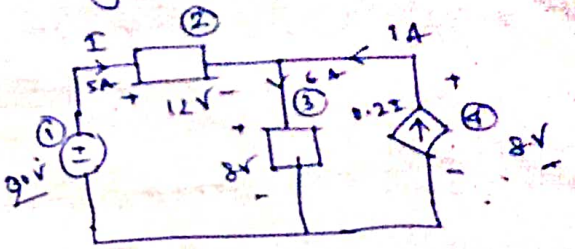
$P = \{(900\pi)\sin(0.18\pi) \times 5\cos(60\pi(0.003))\}$

$= -4500\pi \sin(0.18\pi)\cos(0.18\pi)$

$= -6351$ W. (Ans - (b))

(source delivering/consuming, load delivering/consuming
+ve sign
if vice versa
-ve power)

Q) Calculate the power supplied/absorbed by each element in the given Fig.



$\Rightarrow I_5 = 5A$

$\Rightarrow$

1) power consumed in 20V source: $20 \times 5 = 100$ watt (supply)

2) power consumed in 12 V passive element
$= 12 \times 5A = 60$ watt (absorbed)

3) (apply KCL):
$8 \times (5+I) = 40$ watt (absorbed)

4) Dependent current source:
$1 \times 8 = 8$ watt (supply)

$0.2I = 0.2 \times 5$
$= 1$ A. $I = 5A$.

(those elements which are capable of generating power → Active elements; others passive element

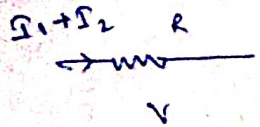
* source ke +ve se current jayegi → supply
load me enter current → +ve P)

Linear elements

follows:

1) principle of supper position (Additive prop)

2) " " Homogeneity (scaling prop)



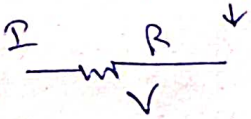
$V_1 = I_1 R$

$V_2 = I_2 R$

$V_{12} = (I_1 + I_2) R$

$= V_1 + V_2$

sup. th



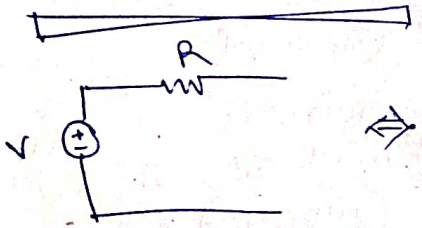
$V = IR$

$V' = (KI)R$

$= KV$

* source I_x :- transformation

direction: from (-) to (+) of voltage



R



$v = iR$

$i = I_M \sin\omega t$

$v = V_m \sin\omega t$

$= R I_m \sin\omega t$

$V_m = R i_m$

ϕ (phase Angle) same

L



$v \propto \frac{di}{dt}$

$v = L \frac{di}{dt}$

$i = i_m \sin\omega t$

$v = L \frac{d}{dt}(i_m \sin\omega t)$

$= L i_m \frac{d}{dt} \sin\omega t$

$= L i_m \omega \cos\omega t$

$= \omega L i_m \cos\omega t$

$= \omega L i_m \left(\sin\omega t + \frac{\pi}{2}\right)$

In Inductor voltage is leading current by $\pi/2$

$v = X_L i_m \left(\sin(\omega t + \frac{\pi}{2})\right)$

$V_m = X_L i_m$

$v = V_m \sin(\omega t + \frac{\pi}{2})$

$\boxed{X_L = \omega L}$

C



$C = \frac{q}{v}$

$q = Cv$

$i = \frac{dq}{dt}$

$= \frac{d}{dt}(Cv)$

$V = V_m \sin(\omega t)$

$i = C \frac{d}{dt}(V_m \sin\omega t)$

$= C V_m \omega (\cos\omega t)$

$= C V_m \omega \left(\sin(\omega t + \frac{\pi}{2})\right)$

$i = \frac{V_m}{X_C} \sin(\omega t + \frac{\pi}{2})$

$= i_m \sin(\omega t + \frac{\pi}{2})$

$I_m = \frac{V_m}{X_C}$

$X_C = \frac{1}{\omega C}$

Resistor

Avg power

$$P = \frac{1}{2\pi}\int_0^{2\pi} v\,i\,d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot i_m \sin\omega t\, d\omega t$$

$\omega t \to \theta$

$$P = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta \cdot i_m \sin\theta\, d\theta$$

$$= \frac{V_m i_m}{2\pi}\int_0^{2\pi} \sin^2\theta\, d\theta$$

$$= \frac{V_m i_m}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\theta}{2}\, d\theta$$

$$= \frac{V_m i_m}{4\pi}\left\{[\theta]_0^{2\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}\right\}$$

$$= \frac{V_m i_m}{4\pi}(2\pi - 0)$$

$$= \frac{V_m i_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}}$$

$\boxed{P = VI}$

V: V_{rms}

I: I_{rms}

P: Avg power

Avg power in ~~Inductor~~ capacitor

$$P = \frac{1}{2\pi}\int_0^{2\pi} v\,i\,d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} I_m V_m \sin\left(\omega t + \frac{\pi}{2}\right)\sin\left(\omega t + \frac{\pi}{2}\right) d\omega t$$

$\omega t \to \theta$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m i_m \sin\left(\theta + \frac{\pi}{2}\right)\sin\left(\theta + \frac{\pi}{2}\right) d\theta$$

$$= \frac{V_m i_m}{2\pi}\int_0^{2\pi} \frac{1-\cos 2\left(\theta+\frac{\pi}{2}\right)}{2}\, d\theta$$

$$= \frac{V_m i_m}{4\pi}\left\{[\theta]_0^{2\pi} - \left[\frac{\sin(2\theta+\pi)}{2}\right]_0^{2\pi}\right\}$$

$$= \frac{V_m i_m}{4\pi}\left\{2\pi - \left(\frac{\sin 4\pi - \sin\pi}{2}\right)\right\}$$

C : current lead

L : voltage lead

capacitor ⇓ $V_m \sin\omega t$ & $I_m \cos\omega t$ ↓ $\left(\sin\left(\omega t + \frac{\pi}{2}\right)\right)$

Inductor ↓ $V_m \left(\sin\omega t + \frac{\pi}{2}\right)$ $= V_m \cos\omega t$ & $I_m \sin\omega t$

Conductor

Avg power:

$$V = V_m \sin\omega t$$

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= i_m \cos\omega t$$

$$P = V_m i_m \sin(\omega t)\cos(\omega t)$$

$$= \frac{V_m i_m}{2}\left(\sin(2\omega t)\right)$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m i_m}{2} \sin(2\omega t)\, d\omega t$$

$$= 0$$

(see next ones)

Inductor

Avg power:-

~~$V = V_m \cos\omega t$~~

~~$I = i_m \cos\left(\omega t - \frac{\pi}{2}\right)$~~

$$V = V_m \sin\omega t \longrightarrow \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = i_m \cos\omega t$$

$$P = V_m i_m \sin\omega t \cos\omega t$$

$$= \frac{V_m i_m}{2}\sin(2\omega t)$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m i_m}{2}\sin(2\omega t)\, d\omega t$$

$$= \frac{V_m i_m}{4\pi}\int_0^{2\pi} \sin(2\omega t)\, d\omega t$$

$$\left[\frac{\cos 2\omega t}{2\omega}\right]_0^{2\pi} = \frac{1}{2}\left[\cos 4\pi - \cos 0\right]$$

$$= \frac{1}{2}[1-1]$$

$$= 0$$

$$= 0$$

Rajiyan Mahammad.

if there is pure inductor and pure capacitor the net energy over a cycle is 0.

R: Energy consuming element.

Boolean Algebra

$\bar{8}\ \bar{4}\ \bar{2}\ \bar{1}$

2's: $(-7) \longrightarrow (\quad)_{2's}$

$+7 \rightarrow 0111 \longrightarrow$ directly 1001

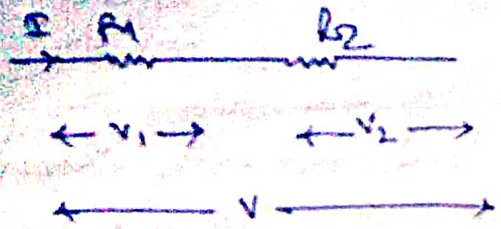
1's $\rightarrow 1000$

$+1$

$+1$

1001

Ckt theory & Networks - 03 (22/09)

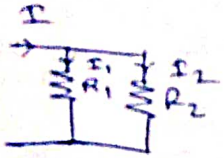
→ 

VDR (voltage division Rule) :-

$$V_1 = V \times \left(\frac{R1}{R1+R2}\right)$$

$$V_2 = V \times \left(\frac{R2}{R1+R2}\right)$$

} same res.

⇒ 

CDR (current division Rule)

$$I_1 = I \times \left(\frac{R2}{R1+R2}\right)$$

$$I_2 = I \times \left(\frac{R1}{R1+R2}\right)$$

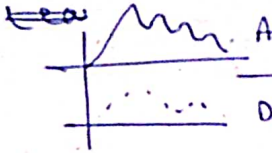
} opp. res.

chapter-2 :- signals :-

A signal is a f/n, representing a physical quantity

signal

- Analog (continuous) (physical) — can't be processed by digital device (as ∞ points in graph) — eg. speech, eg. Temp (continuous graph)
- Digital (form of Discrete signal) — eg. Mobile phones



sampling by ADC machine (converter)

8 bits ($2^8 = 256$ combinations) → dissolution of ADC.

$0 - \frac{5V}{256} \downarrow 0.019$

So,

	output
0 — 0.019	→ 0
0.019 — 0.019 × 2	→ 1
	→ 2
	⋮
× 256	→ 256

eg.

$\frac{2.3464}{0.019}$ → ADC wants 0.019 multiple

$= 123.495$

↓ integer

123×0.019

$= 2.337$

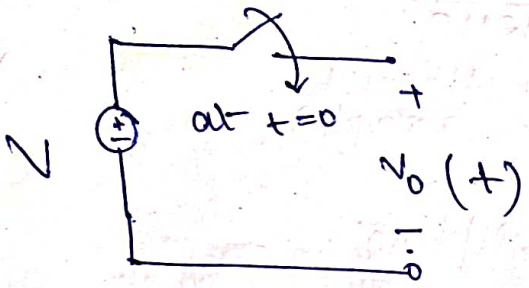
16 | $(123)_{10}$ | 7
16 | 112 | 11 (B)
0

$\therefore (B7)$

2 | 123 | 1
2 | 61 | 1
2 | 30 | 0
2 | 15 | 1

Analog
↓ sampling
Discrete
↓ quantisation
Digital (now processed by any digital devices)

** Unit step function : (USF)

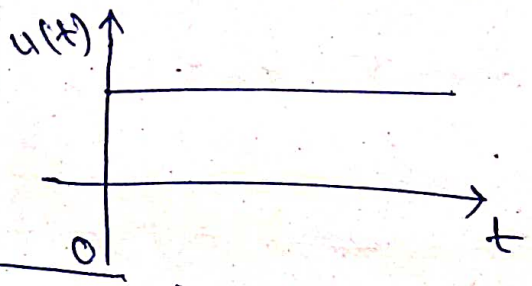


$$v_o(t) = \begin{cases} 0 & \text{for } t<0 \\ V & \text{for } t>,0 \end{cases}$$

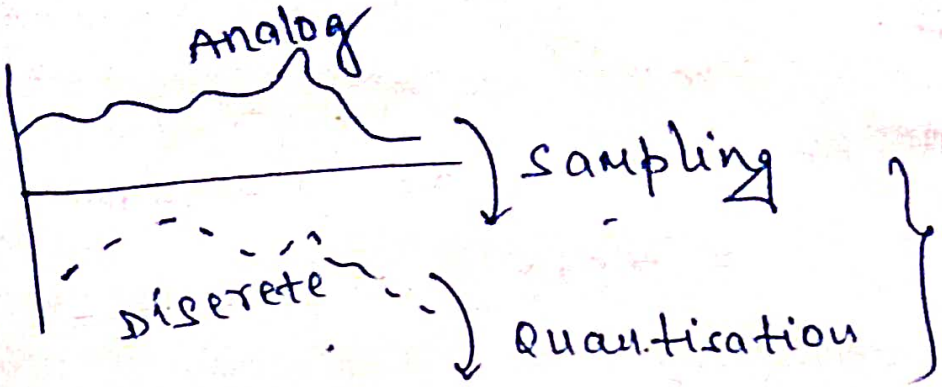
$$\boxed{v_o(t) = V\,u(t)}$$

$$u(t) = \begin{cases} 0 & \text{for } t<0 \\ 1 & \text{for } t>,0 \end{cases}$$

↓
Unit step f/n.



Circuit theory and Networks - 04 (26/07)



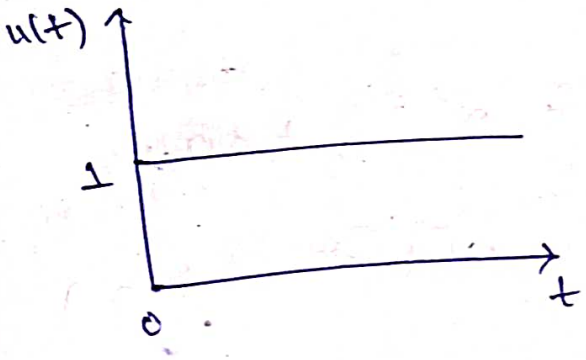
ADC → Analog to digital convertor

values possibilities limited to ADC resolution.

⇒ $\frac{2.3464}{0.019} \rightarrow 123.495$

$123 \times 0.019 =$

$123 \equiv 1111011$ (In devices)

* 

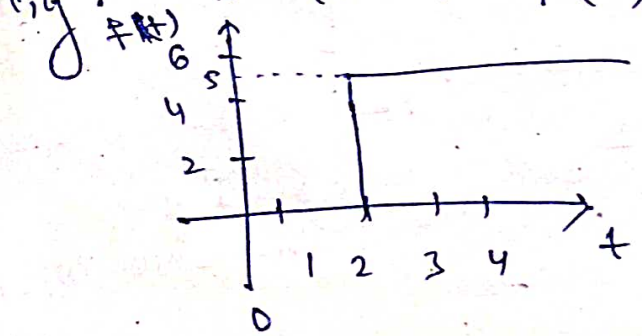
USF

* $f(t) = A\,u(t-a) = \begin{cases} 0 & , t<a \\ A & , t>a \end{cases}$

↓ General step fn

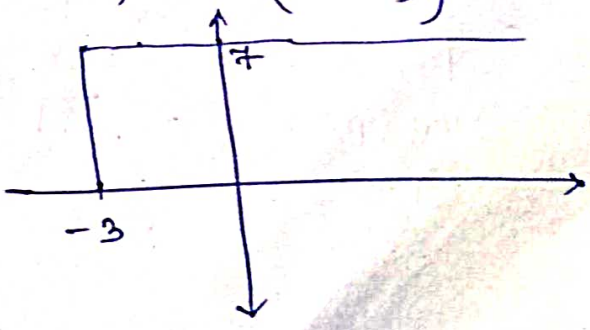
delayed

e.g. $5\,u(t-2) = f(t)$



Advanced

e.g. $f(t) = 7\,u(t+3)$



* Ramp signal / function :-

⇒ For USF :-

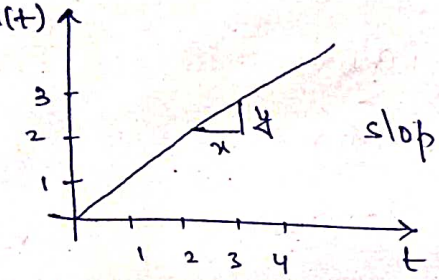
$$\int u(t)\,dt = t \quad \text{as } u(t)=1$$

$$= r(t) \quad \text{or } \int u(t)\,dt = t\,u(t)$$

(multiply of 1 from 0 onwards)

$$r(t) = \begin{cases} 0 & , t<0 \\ t & , t \geq 0 \end{cases}$$

⇒



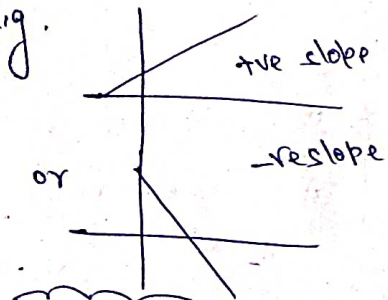
slope = 1.

} Unit ramp f/n as slope = 1

⇒ For RSF :-

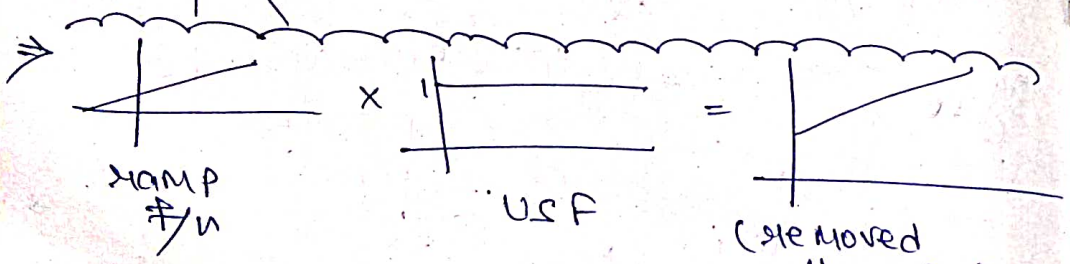
$$f(t) = \begin{cases} 0 & , t<a \\ A(t-a) & , t \geq a \end{cases}$$

e.g.



} General ramp f/n

⇒



Ramp f/n in terms of USF

$$\int v(t)\,dt = t\,u(t)$$

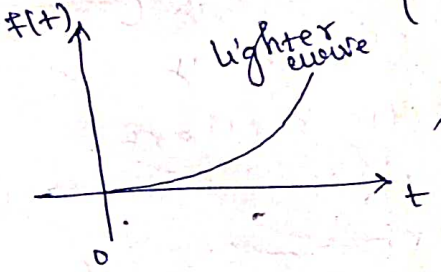
$$= A(t-a)\,u(t-a)$$

* Parabola f/n :-

$$\Rightarrow \iint u(t)\,dt = \begin{cases} 0 & , t<0 \\ \frac{t^2}{2} & , t \geq 0 \end{cases}$$

as $\int t\,dt = \frac{t^2}{2}$

$$\Rightarrow f(t) = \begin{cases} 0 & , t<0 \\ t^2 & , t \geq 0 \end{cases} \quad (\text{remove } 1/2)$$

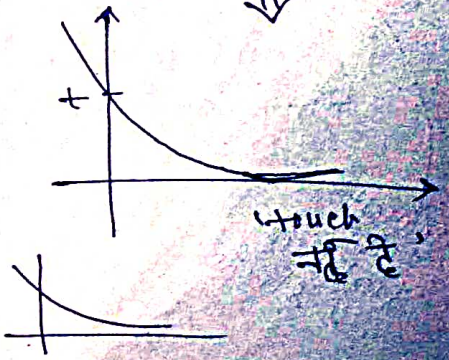


$$\Rightarrow f(t) = \begin{cases} 0 & , t<a \\ A(t-a)^2 & , t \geq a \end{cases}$$

* Exponential f/n :-

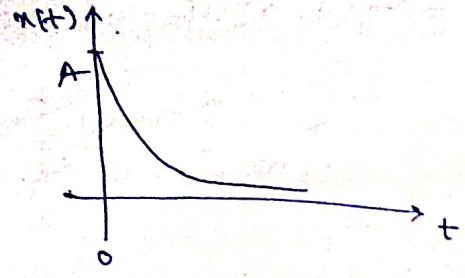
$$f(t) = A e^{-at} = \begin{cases} 0 & , \text{for } t<a \\ A e^{-at} & \text{for } t \geq a \end{cases}$$





exponential f/n in terms of USF

$\Rightarrow x(t) = Ae^{-at}\,u(t) = \begin{cases} 0, & t<0 \\ Ae^{-at} & \text{for } t \geq 0 \end{cases}$

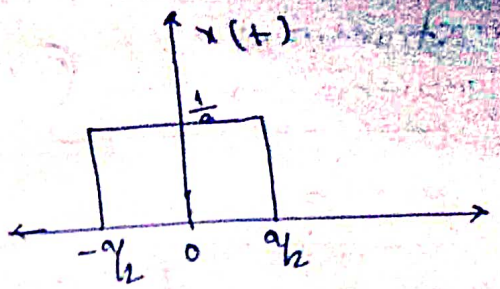


* Note:- theoritically exponential f/n reaches 0 at $t = \infty$
however, practically at $t = 5\tau$ amplitude become negligibly small
where $\tau = \frac{1}{a}$ = time constant.
this τ is the value of t which makes the value of exponent = −1. i.e.
at $t = \tau$
$\Rightarrow -at = -1$
$\Rightarrow t = \frac{1}{a}$, $x(t) = Ae^{-t/\tau}$

* Unit impulse f/n | Dirac delta f/n :-

consider a finite pulse (rectangular signal) $x(t) = \begin{cases} \frac{1}{a} & \forall\ -\frac{a}{2} \leq t \leq \frac{a}{2} \\ 0, & \text{otherwise} \end{cases}$

nearest e.g. lightning strike



Sq. unit

$\Rightarrow$ the area under the pulse is 1 ^ and remains 1 for all the values of a.

the impulse f/n is defined as

$$\delta(t) = \lim_{a \to 0} x(t)$$

$$\Rightarrow \delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

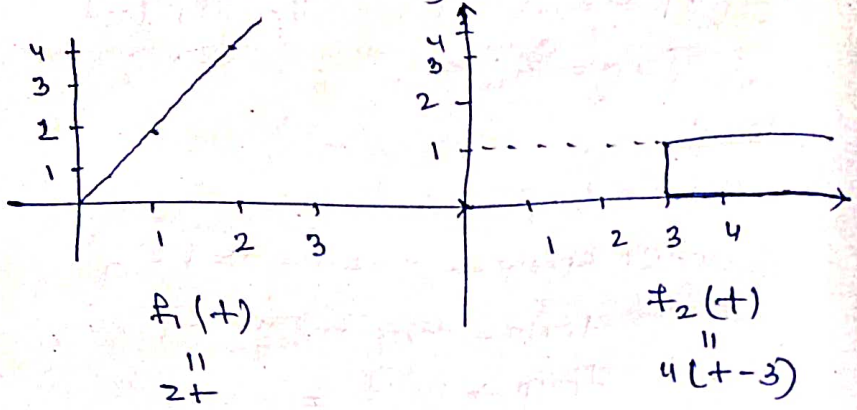
1 Indicates not magnitude but area

unit impulse is a pulse of ∞ height, 0 width and area of 1.

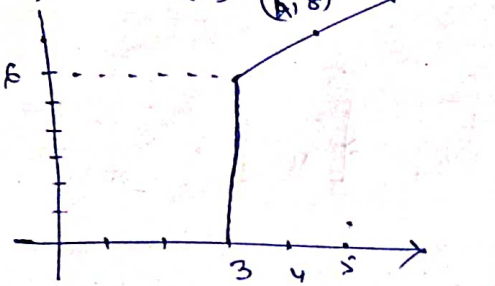
signals :-

e.g. $f(t) = 2t \cdot u(t-3)$

⇒



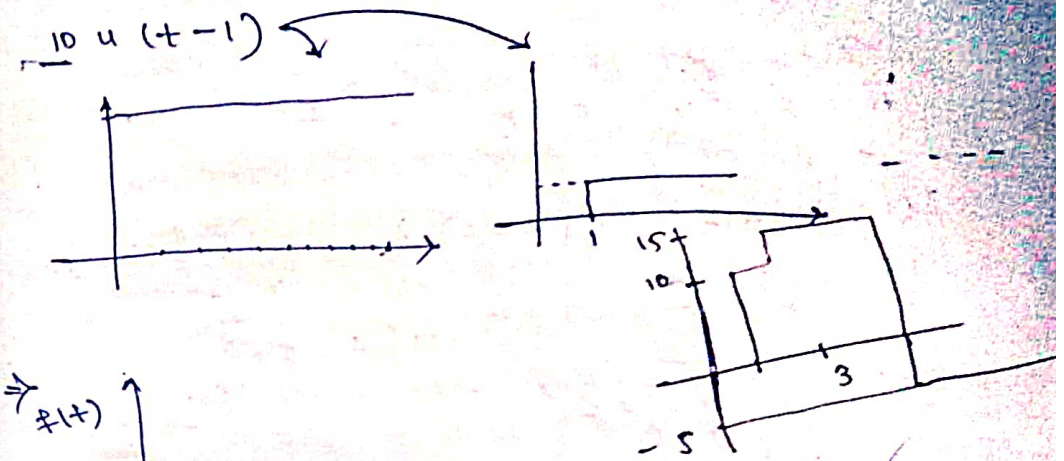
⇒ $f_1(t) \times f_2(t)$



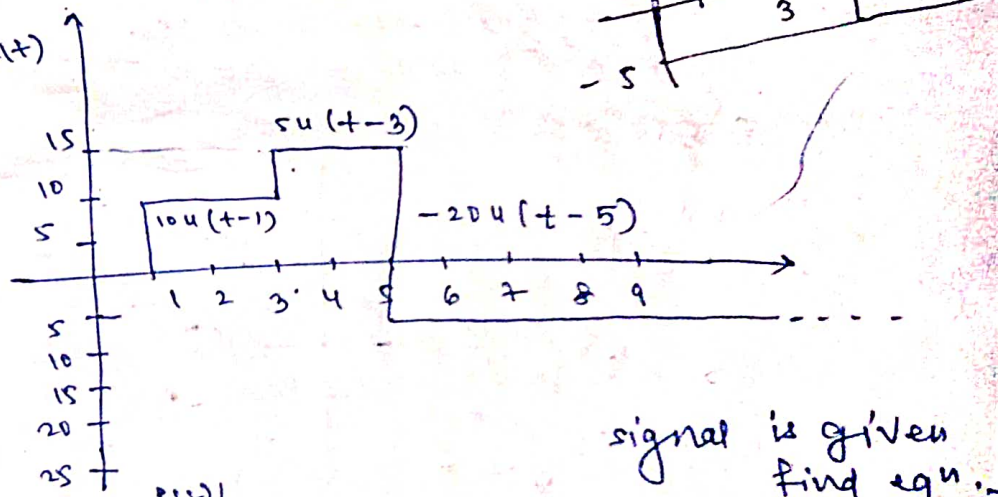
⇒

$t =$	1	2	3	4	5
$f_1(t) =$	2	4	6	8	10
$f_2(t) =$	×	×	×	×	×
	0	0	1	1	1
	0	0	6	8	10

e.g. $f(t) = 10\,u(t-1) + 5\,u(t-3) - 20(u(t-5))$

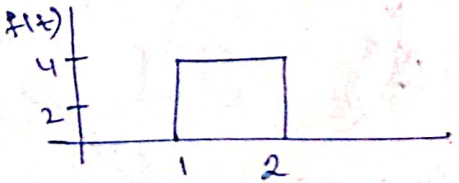


⇒

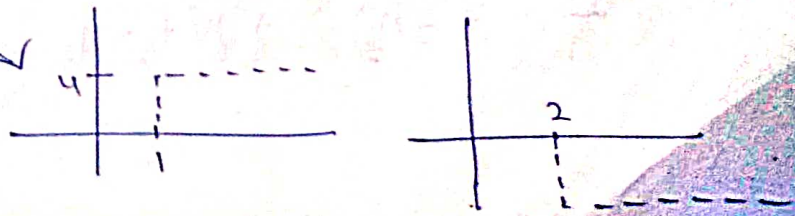


signal is given find eqn :-

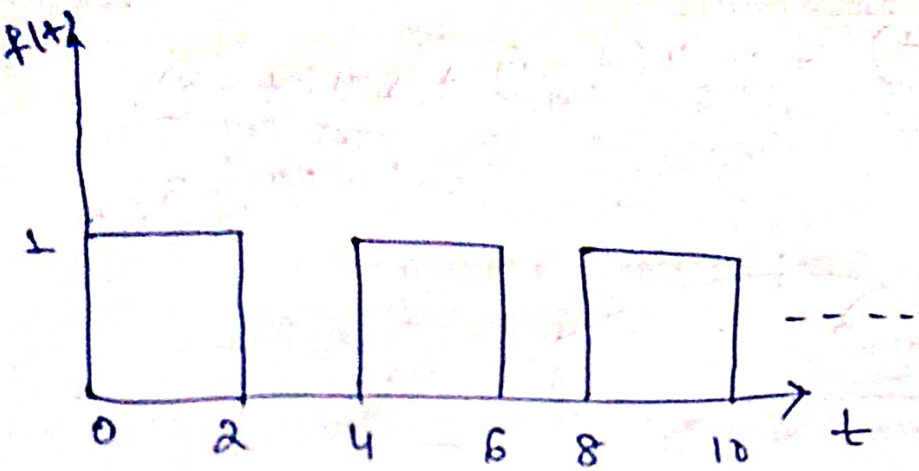
e.g.



$f(t) = 4u(t-1) - 4u(t-2)$



e.g. $f(t)$

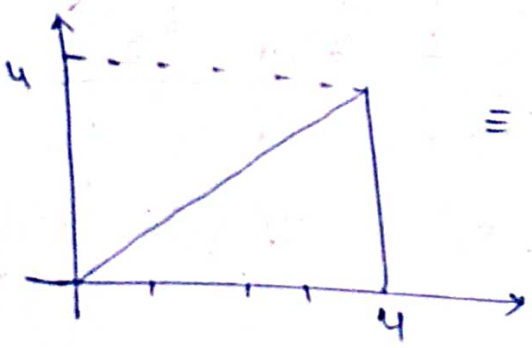


$$f(t) = u(t) - u(t-2) + u(t-4) - u(t-6) \cdots$$

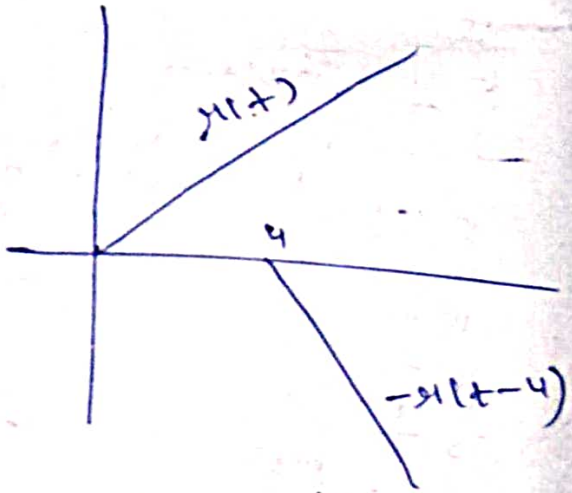
$$= \sum_{n=0}^{\infty} (-1)^n \, u(t-2n)$$

Pulse train
or
clock pulse

e.g.



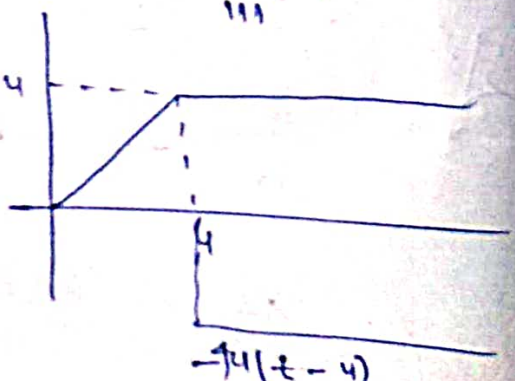
$\equiv$



$f(t)$

$= t\,u(t) - (t-4)(u(t-4)) - 4\,u(t-4)$

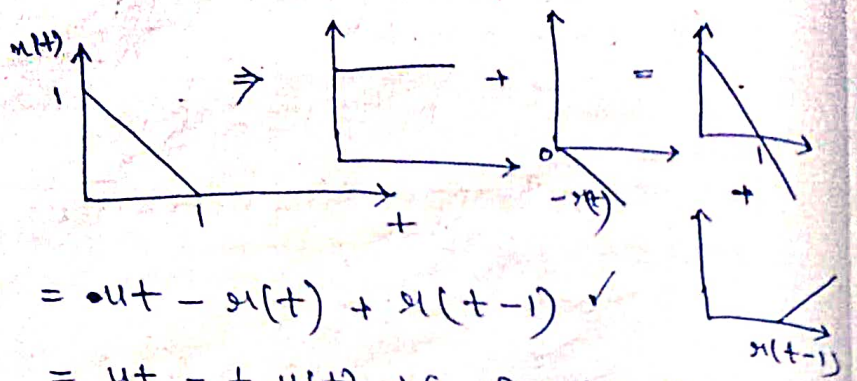
$\equiv$



$= r(t) - r(t-4) - 4\,u(t-4)$

2/8/24 Circuit theory & Networks - 06

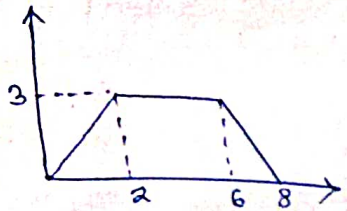
Q1)



$f(t) = u(t) - x(t) + x(t-1)$ ✓

$= u(t) - t\,u(t) + (t-1)\,u(t-1)$

Q1)



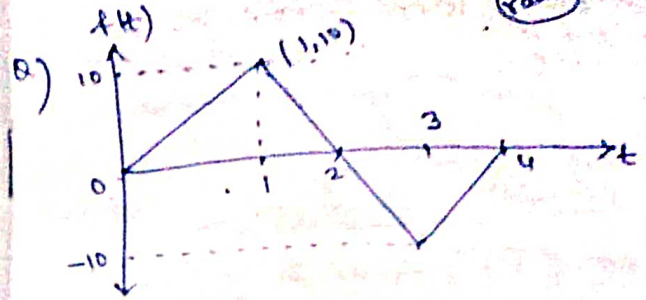
$f(t) = \frac{3}{2}x(t) + \frac{3}{2}x(t-2) - \frac{3}{2}x(t-6) + \frac{3}{2}x(t-8)$

slope

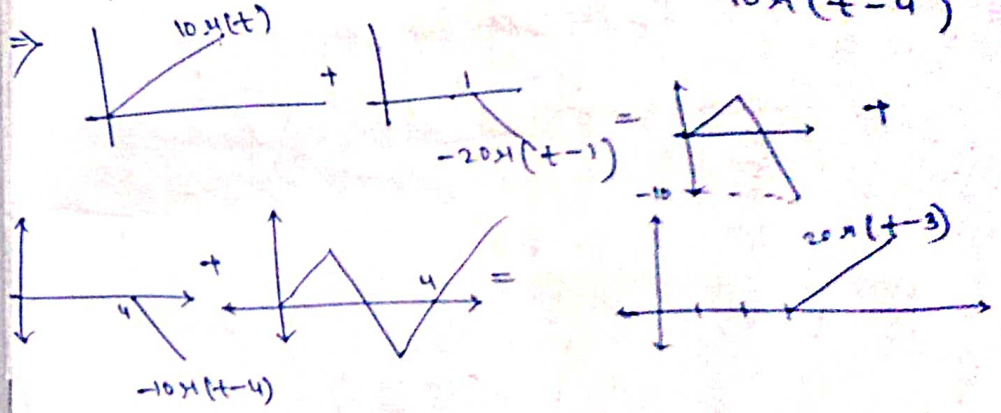
$= \frac{3}{2}\{t\,u(t) - (t-2)u(t-2) - (t-6)u(t-6) + (t-8)u(t-8)\}$

Q) $u(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$

Q)



$f(t) = 10x(t) - 20x(t-1) + 20x(t-3) - 10x(t-4)$



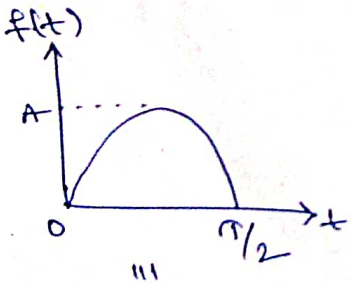
in terms of $y = y_1 + y_2 + y_3$

$y_1 = 10t\,[u(t) - u(t-1)]$

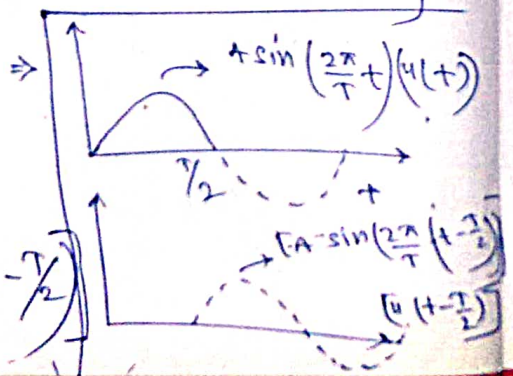
$y_2 = [-10t + 20][u(t-1) - u(t-2)]$

$y_3 = [10(t-4)][u(t-3) - u(t-4)]$

Q)



$f(t) = A\sin\left(\frac{2\pi}{T}t\right)[u(t) - u(t-T/2)]$

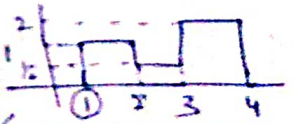


(1,10) (0,0)

$\frac{-10}{-1} = 10$

$\frac{3-10}{2-1} = -7$

$\frac{1}{2} + 10 - 7 = \frac{3}{2}$



$\frac{1}{2}u(t-1) - \frac{1}{2}u(t-2) + \frac{3}{2}u(t-3) - \frac{3}{2}u(t-4)$

Chapter-3 (KVL, KCL ---)

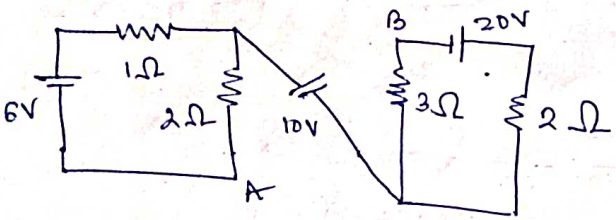
2) * voltage exists even if there is no branch.

* KVL :- Algebraic sum of voltage inside a closed loop is 0.

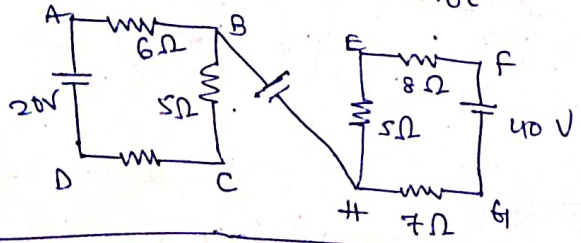
KCL :- $i_{entering} = i_{outgoing}$

Q) find the voltage b/w A & B.

⇒

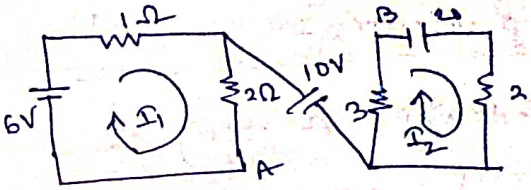


Q) find V_{cf} & V_{Ab}



⇒ Ans) Q1)

Assumptn { entering +ve terminal → +ve, −ve → −ve
current direcn assumptn → +ve, or → −ve



$I_1 + 2I_1 - 6 = 0$

$I_1 = 2$

$V_1 = 2 \times 2 = 4$

$3I_2 - 20 + 2I_2 = 0$

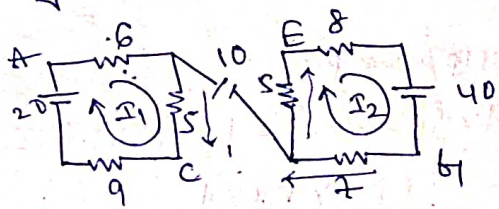
$5I_2 = 20$

$I_2 = 4$

$V_2 = 3 \times 4 = 12$

total voltage $= -4 + 12 + 10 = 18$ V

Ans 2)



$6I_1 + 5I_1 + 9I_1 = 20$

$I_1 = 1$ A

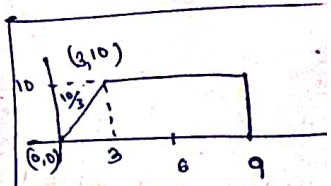
$I_2 = -2$ A

$V_{cE} = -5 + 10 + 10 = -15$ V

$V_{Ab} = 6 \times 1 + 10 + 14 = 30$ V

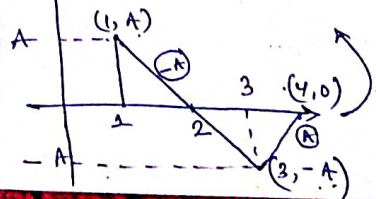
$8I_2 + 40 + 7I_2 + 5I_2 = 0$

$20I_2 = -40$

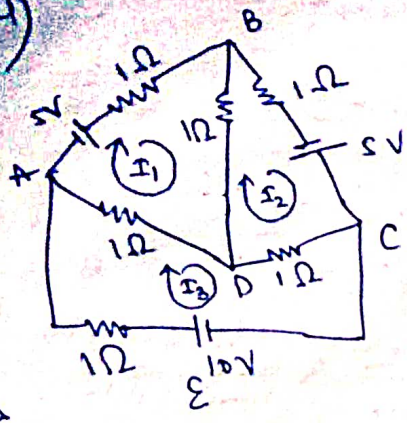


$\frac{10}{3}r(t) - \frac{10}{3}r(t-3) - 10u(t-9) = f(t)$

$f(t) = A\,u(t-1) - A(r(t-1)) + 2A(r(t-3)) - A(r(t-4))$



Q)



Find all the branch current:

$\Rightarrow$

$$I_1 + (I_1 - I_2) + (I_1 - I_3) = 5$$

$$\Rightarrow 3I_1 - I_2 - I_3 = 5 \quad \text{—(1)}$$

$$I_2 + I_2 - I_3 + I_2 - I_1 = 5$$

$$\Rightarrow -I_1 + 3I_2 - I_3 = 5 \quad \text{—(11)}$$

$$I_3 - I_2 + I_3 - I_1 + I_3 = +10$$

$$\Rightarrow -I_1 - I_2 + 3I_3 = 10 \quad \text{—(111)}$$

$$\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 5 \\ 5 \\ 10 \end{vmatrix}$$

$\Delta_2 = 100$
$\Delta_3 = 120$
$I_2 = 6.25\ A$
$I_3 = 7.5\ A$

$$I_1 = \frac{\begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix} \rightarrow \Delta_1}{\begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}} = \frac{5(8) + 1(25) - 1(-35)}{3(8) + 1(-4) - 1(4)}$$

$$= \frac{40 + 25 + 35}{24 - 8} = 6.25\ A$$

$I_{AB} = I_1 = 6.25\ A$

$I_{BC} = I_2 = 6.25\ A$

$I_{AD} = I_3 - I_1 = 7.5 - 6.25 = 1.25\ A$

$I_{CD} = I_2 - I_3 = -1.25\ A$

$I_{AEC} = -I_3 = -7.5\ A$

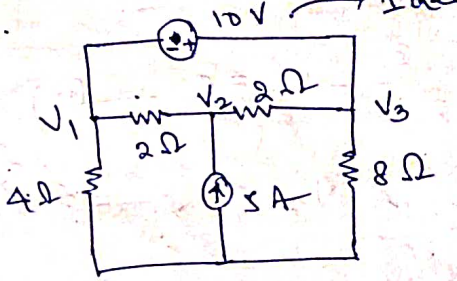
Ideal voltage source

outgoing $\rightarrow$ +ve

in coming $\rightarrow$ -ve

Q2)

Find V_1, V_2, V_3



Next page solution

$$V_3 - V_1 = 10$$

$V_3 = 10 + V_1$
$= 30\ V$

$$\frac{V_1}{4} + \frac{V_1 - V_2}{2} = 0$$

$$3V_1 - 2V_2 = 0 \quad \text{—(1)}$$

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} = 5$$

$$\Rightarrow -V_1 + 2V_2 - V_3 = 10 \quad \text{—(11)}$$

$$\frac{V_3 - V_2}{2} + \frac{V_3}{8} = 0$$

$$\Rightarrow -4V_2 + 5V_3 = 0 \quad \text{—(111)}$$

at (11) $\quad -V_1 + 2V_2 - 10 = -V_1 = 10$

$$\Rightarrow -2V_1 + 2V_2 = 20$$

$$-V_1 + V_2 = 10 \quad \text{—(iv)}$$

$-3V_1 + 3V_2 = 30$
$3V_1 - 2V_2 = 0$

$V_2 = 30\ V$

$3V_1 - 2V_2 = 0$

$2V_2 = 3V_1 \Rightarrow V_1 = \frac{2 \times 30}{3}$

$= 20\ V$

or when there is ideal voltage source b/w two nodes

direct from super node concept

2)

From supernode $\boxed{V_3 - V_1 = 10}$ —(iii)

KCL at supernode (1 & 3) $\frac{V_1}{4} + \frac{V_1 - V_2}{2} + \frac{V_3 - V_2}{2} + \frac{V_3}{8} = 0$ —(i)

$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{2} - 5 = 0$ —(ii)

solve : V_1 & V_2 & V_3

$-V_1 + 2V_2 - V_3 = 10$

$\Rightarrow \underline{2V_1} + \underline{4V_1} - \underline{4V_2} + \underline{4V_3} - \underline{4V_2} + \underline{V_3} = 0$

$\Rightarrow 6V_1 - 8V_2 + 5V_3 = 0$ — (iv)

$V_2 - V_1 + V_2 - V_3 - 10 = 0$

$\Rightarrow -V_1 + 2V_2 - V_3 = 10$ —(v)

putting, $\boxed{V_3 = 10 + V_1}$ in (iv) and (v)

$6V_1 - 8V_2 + 5(10 + V_1) = 0$

$\Rightarrow 11V_1 - 8V_2 = -50$ —(vi)

& $-V_1 + 2V_2 - (10 + V_1) = 10$

$\Rightarrow -2V_1 + 2V_2 = 20$ —(vii)

$\Rightarrow \boxed{-V_1 + V_2 = 10}$

$-8V_1 + 8V_2 = 80$ —(viii), i.e 8 x (vii)

$11V_1 - 8V_2 = -50$

$-11V_1 + 11V_2 = 55$

$3V_2 = 5$

$V_2 = \frac{5}{3}$

now, comparing (vi) & (viii)

$11V_1 - 8V_2 = -50$

$-8V_1 + 8V_2 = 80$

$3V_1 = 30$

$\boxed{V_1 = 10V}$ $V_1 = \frac{10}{3}$ V

$\boxed{V_3 = 20V}$ $V_3 = 10 = \frac{10}{3}$

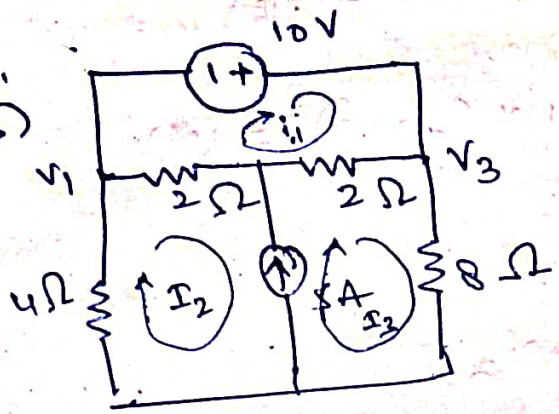
$V_2 = 5 + V_1$

$\boxed{V_2 = 15V}$ $V_2 = 5 - \frac{10}{3} = \frac{20}{3} = \frac{5}{3}$

$\boxed{V_1 = 10V}$
$\boxed{V_2 = 20V}$
$\boxed{V_3 = 20V}$

circuit theory - 08 (09/08/2024)

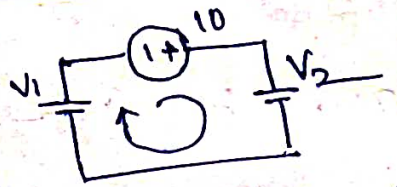
Q) (prev. class)



Final Ans

$V_1 = 10\ V$

$V_2 = V_3 = 20\ V$



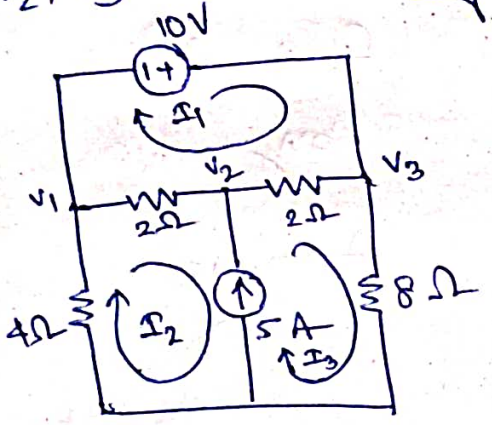
$-V_1 - 10 + V_3 = 0$

$V_3 - V_1 = 10$

solve super node using this technique.

solve the above ckt using mesh analysis:-

find V_1, V_2, V_3



Loop-1-KVL

$-10 + 2(I_1 - I_3) + 2(I_1 - I_2) = 0$

$\Rightarrow 4I_1 - 2I_2 - 2I_3 = 10$

$\Rightarrow 2I_1 - I_2 - I_3 = 5 \quad -(1)$

when there is ideal current source b/w two nodes/loops

~~Loop-2~~ KVL in supermesh (1 & 2)

$4I_2 + 2(I_2 - I_1) + 2(I_3 - I_1) + 8I_3 = 0$

$\Rightarrow -4I_1 + 6I_2 + 10I_3 = 0$

$\Rightarrow -2I_1 + 3I_2 + 5I_3 = 0 \quad -(11)$

By supermesh eqn

$I_3 - I_2 = 5 \quad -(111) \Rightarrow \boxed{I_3 = 5 + I_2}$

$\Rightarrow 2I_1 - I_2 - 5 - I_2 = 5$

$\Rightarrow 2I_1 - 2I_2 = 10 \Rightarrow \boxed{I_1 - I_2 = 5}$

$-2I_1 + 3I_2 + 25 + 5I_2 = 0$

$\Rightarrow -2I_1 + 8I_2 = -25$

$2I_1 - 2I_2 = 10$

$6I_2 = -15$

$I_2 = -2.5$

$I_1 = 2.5$

$I_3 = 2.5$

Now, calculating nodal voltages

$V_1 = -4I_2 = -4 \times (-2.5) = 10V$

$V_2 = 8I_3 = 20V$

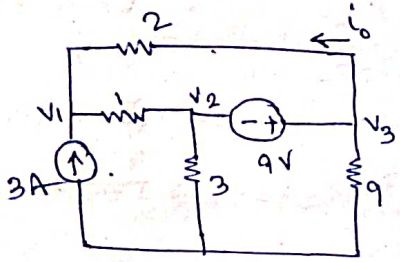
$V_2 = 2(I_3 - I_1) + V_3$

$= 2(2.5 - 2.5) + V_3$

$= 0 + 20$

$= 20\ V$

Q2)



Find i_o by node Analysis :-

⇒ KCL at node-1 :

$$-3 + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0$$

⇒ $3V_1 - 2V_2 - V_3 = 6 \quad -(1)$

⇒ KCL at ~~A~~ super node (2 & 3) :-

$$\frac{V_2 - V_1}{1} + \frac{V_2}{3} + \frac{V_3 - V_1}{2} + \frac{V_3}{9} = 0$$

⇒ $18V_2 - 18V_1 + 6V_2 + 9V_3 - 9V_1 + 2V_3 = 0$

⇒ $-27V_1 + 24V_2 + 11V_3 = 0 \quad -(11)$

FROM node 2 & 3

$0V_1 - V_2 + V_3 = 9 \quad -(111)$

$V_1 = 19/2 = 9.5\ V$ ✓

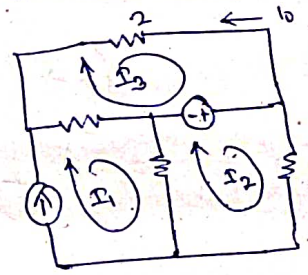
$V_2 = 9/2 = 4.5\ V$ ✓

$V_3 = 27/2 = 13.5\ V$ ✓

$i_o = \frac{V_3 - V_1}{2} = \frac{13.5 - 9.5}{2} = 2A$

$$\begin{bmatrix} 3 & -2 & -1 \\ -27 & 24 & 11 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 9 \end{bmatrix}$$

* solve the above Q's using Mesh Analysis Find i_o



$I_1 = 3\ A$

$3(I_2 - I_1) - 9 + 9I_2 = 0$

$2I_3 + 9 + I_3 - I_1 = 0$

⇒ $12I_2 - 3I_1 = 9$

⇒ $4I_2 - I_1 = 3$ ⇒ $I_2 = 6/4 = 3/2$

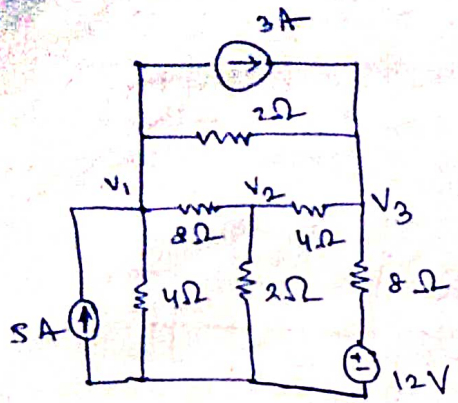
$3I_3 - I_1 = -9$ $= 1.5\ A$

$I_2 = 1.5A$

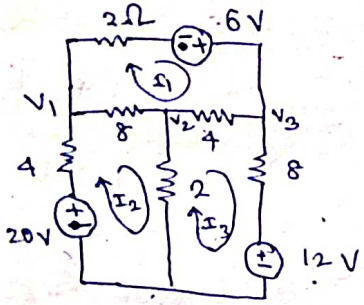
$3I_3 = -6$

$I_3 = -2\ A$ | $i_o = +2\ A$

Q3)



Find V_1, V_2, V_3 using Mesh Analysis





$2I_1 + 6 + (I_1 - I_3)4 + 8(I_1 - I_2) = 0$

$\Rightarrow 14I_1 - 8I_2 - 4I_3 = +6$

$\Rightarrow \boxed{7I_1 - 4I_2 - 2I_3 = +3}$ ——(i)

$8(I_2 - I_1) + 2(I_2 - I_3) + 20 + 4I_2 = 0$

$\Rightarrow -8I_1 + 14I_2 - 2I_3 = -20$

$\Rightarrow \boxed{-4I_1 + 7I_2 - I_3 = +10}$ ——(ii)

$4(I_3 - I_1) + 8I_3 + 12 + 2(I_3 - I_2) = 0$

$\Rightarrow -4I_1 - 2I_2 + 14I_3 = -12$

$\Rightarrow \boxed{-2I_1 - I_2 + 7I_3 = -6}$ ——(iii)

$I_1 = \frac{28}{15} = 1.8666$

$I_2 = \frac{5}{2} = 2.5$

$I_3 = \frac{1}{30} = 0.0333$

$\boxed{-4I_2 + 20 = V_1}$

$V_1 = 10\ V$

$\boxed{8 \times I_3 + 12 = V_3}$

$V_3 = 12 + 8 \times 0.0333$

$= 12.26664\ V$

$V_2 = 4 \times (I_3 - I_1) + 12 \cdot 2666$

$= 4 \times$

$\boxed{V_2 = 2(I_2 - I_3)}$

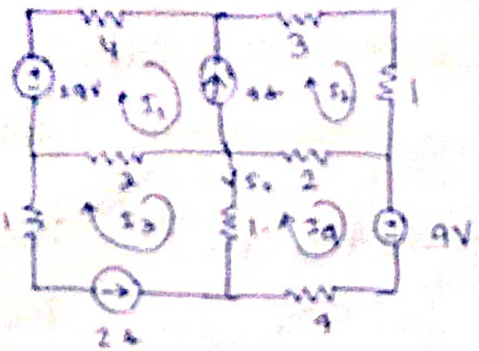
$= 2 \times 2.4667$

$= 4.9334\ V$

$\boxed{V_1 = 10V}$
$\boxed{V_2 = 4.9334\ V}$
$\boxed{V_3 = 12.26664}$

Ans.

04)



$4I_1 - 24 + 3I_2 + I_2 + 2(I_2 - I_4) + 2(I_1 - I_3) = 0$

$\Rightarrow 6I_1 + 6I_2 - 2I_3 - 2I_4 = 24$

$\Rightarrow 3I_1 + 3I_2 - I_3 - I_4 = 12$

$\Rightarrow 3I_1 + 12 + 3I_1 + 2 - I_4 = 12$

$I_3 = -2$ | $I_2 - I_1 = 4 \Rightarrow I_2 = 4 + I_1$

$2(I_4 - I_2) + 9 + 4I_4 + (I_4 - I_3) = 0$

$\Rightarrow 6I_1 - I_4 = -2 \quad -(I)$

$\Rightarrow 2I_4 - 2I_2 + 9 + 4I_4 + I_4 - I_3 = 0$

$\Rightarrow 7I_4 - 8 - 2I_1 + 2 + 9 = 0$

$\Rightarrow -2I_1 + 7I_4 = -3 \quad -(II)$

$42I_1 - 7I_4 = -14$

$-2I_1 + 7I_4 = -3$

$40I_1 = -17$

$I_1 = -\frac{17}{40}$

$I_4 = -2 + 6 \times \frac{17}{40}$

$= \frac{102}{40} - 2$

$= \frac{22}{40} = \frac{11}{20}$

$= 6 \times \left(-\frac{17}{40}\right) + 2$

$= -0.55 \text{ A}$

$I_0 = (I_3 - I_4)$

$= (-2) - (0.55)$

$= -2 + 0.55$

$= -1.454$

20) $\frac{110 \times 55}{100}$ D.

$6(34) + 2(-17) - 2(9)$

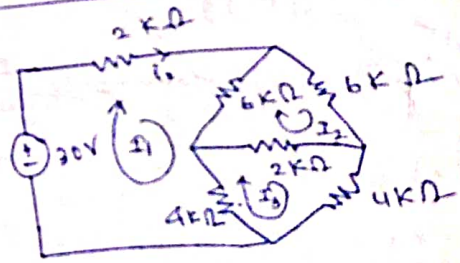
$= 204 - 53 - 14$

$= 204 - 67$

$= \frac{204}{(3 \times 67)} = \frac{204}{201} = 1.89$

$= \frac{15}{(3 \times 17)} \times \frac{34}{17} = \frac{30}{119} = 4.28 \text{ mA}$

05)



Find i_0 using mesh analysis.

$2(I_1) + 6(I_1 - I_2) + 4(I_1 - I_3) = 30$

$\Rightarrow 6I_1 - 3I_2 - 2I_3 = 15 \quad -(I)$

$6I_2 + 2(I_2 - I_3) + 6(I_2 - I_1) = 0$

$\Rightarrow -3I_1 + 7I_2 - I_3 = 0 \quad -(II)$

$4I_3 + 4(I_3 - I_1) + 2(I_3 - I_2) = 0$

$\Rightarrow -2I_1 - I_2 + 5I_3 = 0 \quad -(III)$

$\begin{bmatrix} 6 & -3 & -2 \\ -3 & 7 & -1 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}$

$I_1 = 4.28 \text{ mA}$

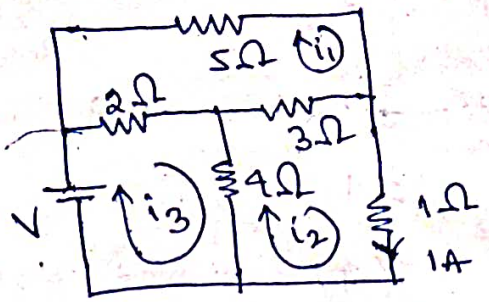
$I_2 = 2.14 \text{ mA}$

$I_3 = 2.14 \text{ mA}$

$I_0 = 4.28 \text{ mA}$

circuit theory - 09 (12/08)

Q) Determine source voltage V (: Both Mesh & nodal Analysis)



$$10i_1 - 3i_2 - 2i_3 = 0 \quad \text{—(I)}$$
$$-2i_1 - 4i_2 + 6i_3 = +V \quad \text{—(III)}$$
$$-3i_1 + 8i_2 - 4i_3 = 0 \quad \text{—(II)}$$

$$\boxed{i_2 = 1\ A}$$

$$10i_1 - 3 - 2i_3 = 0$$

$$10i_1 - 2i_3 = 3 \quad \text{—(I)}$$
$$-3i_1 - 4i_3 = -8 \quad \text{—(II)}$$
$$-2i_1 + 6i_3 = 4 + V \quad \text{—(III)}$$

$$\begin{vmatrix} 10 & 0 & -2 \\ -3 & 0 & -4 \\ -2 & 0 & 6 \end{vmatrix}$$

$$20i_1 - 4i_3 = 6$$
$$-3i_1 - 4i_3 = -8$$
$$23i_1 = 14$$

$$\boxed{i_1 = \frac{14}{23}}$$

$$2i_3 = 10i_1 - 3$$
$$i_3 = \frac{10 \times \frac{14}{23} - 3}{2}$$
$$= \frac{\frac{140}{23} - 69}{2 \times 23}$$
$$= \frac{\frac{3151}{46}}{46} = 68.5$$

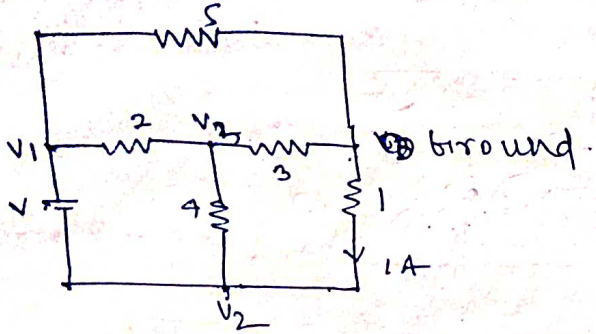
$$\frac{71}{46} = 1.54\ A$$

$$\boxed{i_3 = 1.54}$$

$$-\frac{28}{23} - 4 + \frac{71 \times 6}{46} = +V$$

$$\boxed{V = +4.050\ V}$$

Node analysis :-



$$\frac{V_1}{5} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} + \frac{V_2}{1} = 0 \quad \text{—(1)}$$

$$V_1 - V_2 = V$$

$$\frac{V_3 - V_1}{2} + \frac{V_3 - V_2}{4} + \frac{V_3}{3} = 0 \quad \text{—(II)}$$

$$4V_1 + 10V_1 - 10V_3 + 5V_2 - 5V_3 + 20V_2 = 0$$

$$\Rightarrow 14V_1 + 25V_2 - 15V_3 = 0 \quad \text{—(1)}$$

$$6V_3 - 6V_1 + 3V_3 - 3V_2 + 4V_3 = 0$$

$$-6V_1 - 3V_2 + 13V_3 = 0 \quad \text{—(II)}$$

$V_2 = -1\ V$

as $\frac{V_2 - 0}{1} = -1$

$$\Rightarrow 14V_1 - 15V_3 = 25$$

$$-6V_1 + 13V_3 = -3$$

$$14 \times 13 V_1 - 15 \times 13 V_3 = 25 \times 13$$

$$-6 \times 15 V_1 + 13 \times 15 V_3 = -3 \times 15$$

$$92V_1 = 280$$

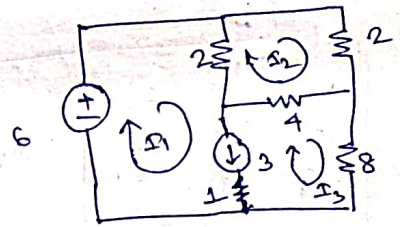
$$V_1 = \frac{280}{92} = 3.04\ V$$

$$V = V_1 - V_2$$

$$= 3.04 + 1$$

$$= 4.04\ V$$

Q) Find I_1, I_2, I_3 (using mesh analysis)



$$I_1 - I_3 = 3 \quad \text{—(III)}$$

$$2(I_1 - I_2) - 6 + 8I_3 + 4(I_3 - I_2) = 0 \quad \text{—(I)}$$

$$2(I_2 - I_1) + 2I_2 + 4(I_2 - I_3) = 0 \quad \text{—(II)}$$

$$2I_1 - 2I_2 + 8I_3 + 4I_3 - 4I_2 = 6$$

$$\Rightarrow 2I_1 - 6I_2 + 12I_3 = 6$$

$$\Rightarrow I_1 - 3I_2 + 6I_3 = 3 \quad \text{—(I)}$$

$$2I_2 - 2I_1 + 2I_2 + 4I_2 - 4I_3 = 0$$

$$\Rightarrow -2I_1 + 8I_2 - 4I_3 = 0$$

$$\Rightarrow -I_1 + 4I_2 - 2I_3 = 0 \quad \text{—(II)}$$

$$\begin{vmatrix} 1 & -3 & 6 \\ -1 & 4 & -2 \\ 1 & 0 & -1 \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \\ I_3 \end{vmatrix} = \begin{vmatrix} 3 \\ 0 \\ 3 \end{vmatrix}$$

$1(-4) + 3(3) + 6(-4)$

$= -4 + 9 - 24$

$= -19 = \Delta$

$I_1 = \frac{66}{19}$

$= 3.47\ A$

$I_1 = 3.47\ A$

$I_2 = 1.105\ A$

$I_3 = 0.473\ A$

$$\begin{vmatrix} 3 & -3 & 6 \\ 0 & 4 & -2 \\ 3 & 0 & -1 \end{vmatrix} = \Delta_x$$

$= 3(-4) + 3(6 - 24)$

$= -12 - 54$

$= -66 = \Delta_x$

$$\Delta_y = \begin{vmatrix} 1 & 3 & 6 \\ -1 & 0 & -2 \\ 1 & 3 & -1 \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} 1 & -3 & 3 \\ -1 & 4 & 0 \\ 1 & 0 & 3 \end{vmatrix}$$

$= -3(3) - 3(4) = -9 - 12 = -21$

$= 1(6) - 3(3) + 6(-3)$

$= 6 - 9 - 18 = -21 = \Delta_y$

$= 3(-4) + 3(1)$

$= -12 + 3 = -9$

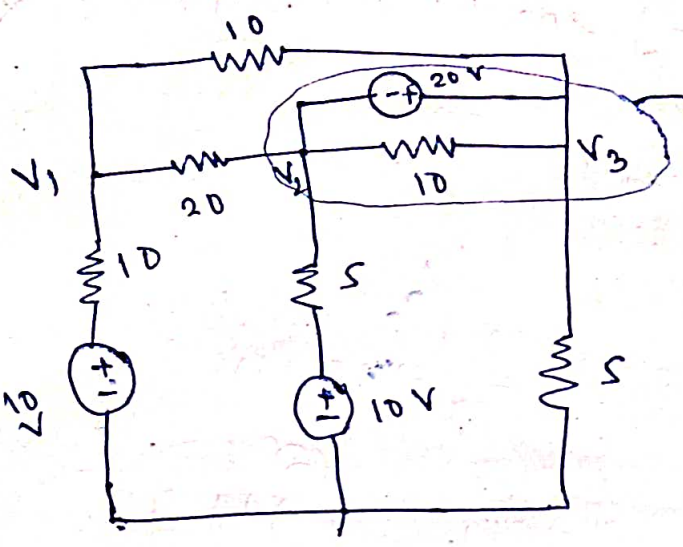
$I_2 = \frac{-21}{-19} = \frac{21}{19} = 1.1052\ A$

$I_3 = \frac{9}{19} = 0.473\ A$

$= 1(12) + 3(-3) + 3(-4)$

$= 12 - 9 - 12 = -9$

Q) Find V_1, V_2, V_3



⇒ super node

$V_3 - V_2 = 20$ —(I)

⇒ From node 1.

$$\frac{V_1 - V_2}{20} + \frac{V_1 - 10}{10} + \frac{V_1 - V_3}{10} = 0$$

$$\Rightarrow \quad 5V_1 - V_2 - 2V_3 = 20 \quad —(II)$$

⇒ From supernode node :-

$$\frac{V_2 - 10}{5} + \frac{V_3}{5} + \frac{V_2 - V_1}{20} + \frac{V_3 - V_1}{10} = 0$$

$$\Rightarrow \quad -3V_1 + 5V_2 + 6V_3 = 40 \quad —(III)$$

Now, replacing $\boxed{V_3 = 20 + V_2}$ in (II) & (III)

we get from (II) $\boxed{5V_1 - 3V_2 = 60}$ —(IV)

we get from (III) $\boxed{-3V_1 + 11V_2 = -80}$ —(V)

solving (IV) & (V) we get, $\boxed{V_2 = -4.78\,V}$, $\boxed{V_3 = 15.21\,V}$

From (IV) we get, $\boxed{V_1 = 9.13\,V}$

AC - circuits

* Polar form ⇒ $|A| \angle \theta$ (magnitude, angle) — magnitudes (×) or (÷); angle (+) or (−)

⇓

Multiplication & division

* Rectangular form ⇒ $x + jy$ — Add & substract.

⇒ calculator:-

e.g. $3 + j4$ →

POl(

POl(3, 4)

enter [=] 5

RCL + tan(F)

= 53.13° ✓

In your calculator direct conversion b/w Rec & POl

⇒

Rec(

[=] → Rec(5, 53.13)

3.0000

then → RCL + tan(F)

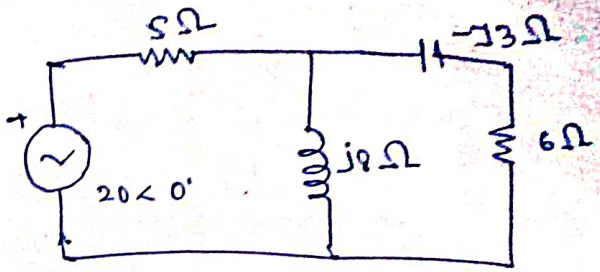
3.999

3 + 4j

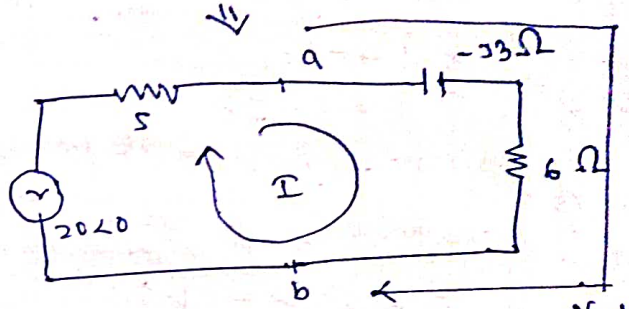
when −ve number (−) use

Determine the current through & voltage across the inductor in the given ckt using thevenin's theorem:-

⇒



⇓



$$I = \frac{20\angle 0^\circ}{5 - j3 + 6} = \frac{20}{11 - j3}$$

(use calculator)

$$= \frac{20}{11.4\,(-15.25)}$$

$$= 1.75 \angle 15.25 \text{ A}$$

$$5I - j3I + 6I - 20\angle 0 = 0$$

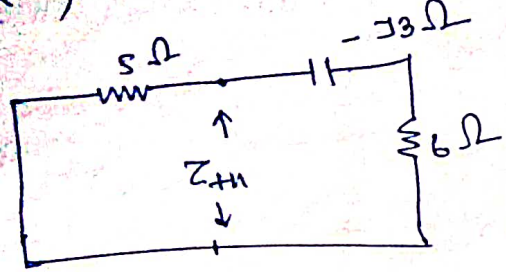
$$I = \frac{20\angle 0^\circ}{5 + 6 - j3} = \frac{20\angle 0^\circ}{11 - j3}$$

V = voltage drop

$(6 - j3)I = (6.71 \angle -26.56) \times (1.75 \angle 15.25)$

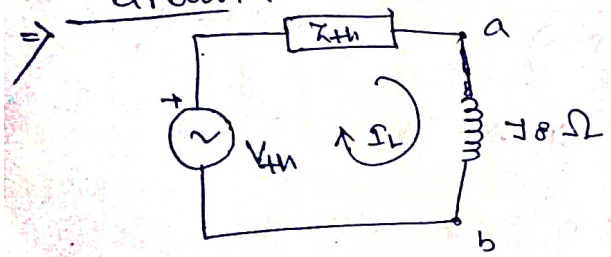
$V_{ab} = 11.74 \angle -11.31^\circ$ V

Equivalence Impedence :-
(Z_{th})



$$Z_{th} = 5 \,||\, 6-j3$$
$$= \frac{5(6-j3)}{5+6-j3}$$
$$= \frac{5(6.71 \angle -26.56)}{11.4 \angle -15.25}$$
$$= \frac{33.55 \angle -26.56}{11.4 \angle -15.25}$$
$$= 2.94 \angle -11.31\ \Omega$$

thevenin's equivalent circuit :-



$$I_L = \frac{V_{th}}{Z_{th}+j8} = \frac{11.74 \angle -11.31}{(2.94 \angle -11.31)+j8}$$
$$= \frac{11.74 \angle -11.31}{2.88 - j0.576 + j8}$$
$$= \frac{11.74 \angle -11.31}{2.88 + j7.424} = \frac{11.74 \angle -11.31^\circ}{7.94 \angle 68.79}$$

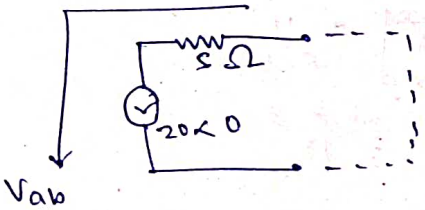
$$V_L = I_L (j8)$$
$$= (1.474 \angle -80.1)(j8)$$
$$= (1.474 \angle -80.1)(8 \angle 90)$$
$$= \cancel{9.474 \angle 9.9^\circ}$$
$$= 11.792 \angle 9.9^\circ \ V$$

current through the Inductor
$= I_L = 1.474 \angle -80.1^\circ$ A.

voltage across the Inductor
$= V_L = 11.792 \angle 9.9^\circ$ V

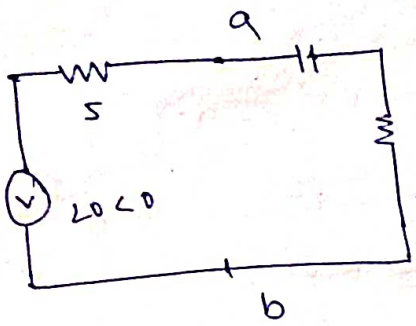
I_L = $1.474 \angle -80.1^\circ$ A
V_L = $11.792 \angle 9.9^\circ$ V

*



$$V_{ab} = -5I + 20$$
$$= -5(1.75 \angle 15.25) + 20$$
$$= -(8.75 \angle 15.25) + 20$$
$$= -[8.44 + j2.3] + 20$$
$$= -8.44 - j2.3 + 20$$
$$= 11.56 - j2.3$$
$$= 11.78 \angle -11.25^\circ$$

By Node Analysis



$$\frac{V-20}{5} + \frac{V}{6-j3} = 0$$

$$\Rightarrow V\left[\frac{1}{5} + \frac{1}{6-j3}\right] = 4$$

$$\Rightarrow V\left[\frac{11-j3}{5(6-j3)}\right] = 4$$

$$\Rightarrow V\left[\frac{11-j3}{30-15j}\right] = 4$$

$$V\left[\frac{11.40\angle -15.25}{33.54\angle -26.56}\right] = 4$$

$$V\,[0.34\angle 11.31] = 4$$

$$V = \frac{4}{0.34}\angle -11.31$$

$$= 11.76\angle -11.31$$

$$V = 4\left(\frac{2.94\angle -18.56}{}\right)$$

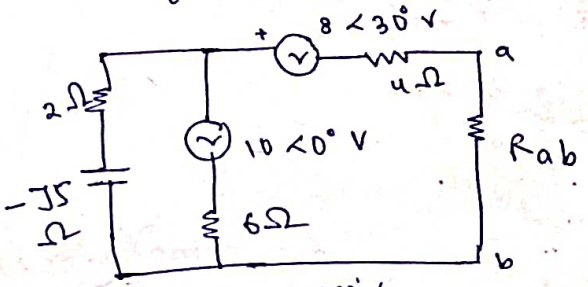
$$= 11.76\angle -11.31°$$

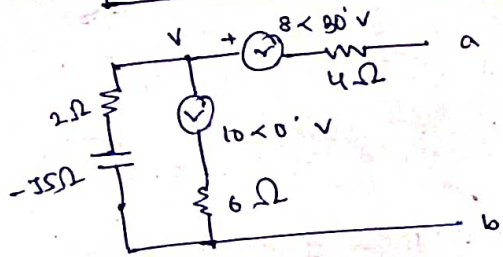
$$= 4(2.62 - 1.31j)$$

$$= 10.48 - 5.24j$$

$$= 11.71\angle -26.56$$

Q1) Determine the variation of current in Rab when it is changed from 2Ω to 10Ω (thevenin's / Norton's)





$$\frac{V}{2-j5} + \frac{V-10}{6} = 0 \quad \text{or} \quad I = \frac{10}{8-j5}$$

$$V\left[\frac{1}{2-j5} + \frac{1}{6}\right] = \frac{10}{6} \Rightarrow V\left(\frac{8-j5}{6(2-j5)}\right) = \frac{10}{6}$$

$$\Rightarrow V = \frac{10}{6} \times \frac{6(2-j5)}{8-j5}$$

$$= \frac{20-100j}{8-j5} = \frac{101\angle -78.69°}{9.43\angle -32°}$$

$$= 90.57\angle -46.69 \qquad = 10.60\angle -46.69$$

$$V_{ab} = -8\angle 30° + 90.57\angle -46.69$$

$$= -6.92 - 4j + 62 - 65.90j$$

$$= 55.08 - 69.90j$$

$$= 89\angle -51.76°$$

From mesh

$$I = \frac{10}{8-3j} = 1.06 \angle 32^\circ \text{ A}$$

$V_{ab} = -8\angle 30 + 10\angle 0^\circ - 6I$

$\Rightarrow V_{ab} = -8\angle 30^\circ + (2-3j)I$

$= -8\angle 30^\circ + (5.38\angle -68.19^\circ)(1.06\angle 32^\circ)$

$= -8\angle 30^\circ + 5.70\angle -36.19$

$= -6.92 - 4j + 4.6 - 3.36j$

$= -12.31 + 10 - 7.36j$

$= -(2.31 + j7.36)$

$= -7.724 \angle 72.6$ (Ans)

$\Rightarrow V = 5.73 \angle -36.2$

$V_{ab} = -8\angle 30 + V$

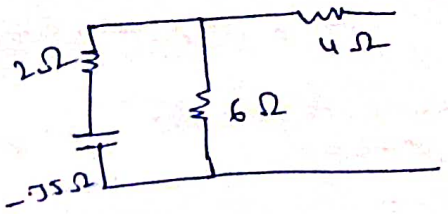
$= -6.92 - j4 + 4.62 - j3.38$

$= -2.3 - 7.38$

$= -7.73 \angle -107.35^\circ$

---

* source will be replaced by short ckt *



$2 - j3 \,||\, 6 + 4$

$\Rightarrow \frac{(2-3j)\times 6}{8-3j} + 4$

$= \frac{12-30j}{8-3j} + 4$

$= \frac{-32.31\angle -68}{9.4\angle -32} + 4$

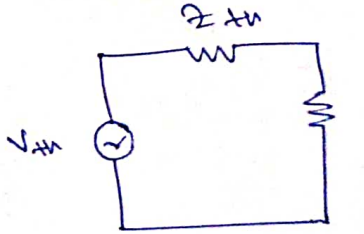
$= 3.43\angle -36 + 4$

$= 2.77 - 2j + 4$

$= 6.77 - 2j$

$= 7.05 \angle -16.45\ \Omega$ (Ans)

68
32
36



$I_L = \frac{V_{th}}{Z_{th} + R_{ab}}$

when $R_{ab} = 2\Omega \rightarrow I_L = \frac{-7.73\angle 72.6}{6.77 - j2 + 2}$

$\Rightarrow I_2 = -0.86\angle 85.446$ A

$= +0.86\angle -94.5$ A

$= \frac{-7.73\angle 72.6}{8.77 - j2}$

$= \frac{-7.73\angle 72.6}{9\angle 12.85}$

$R_{ab} = 10\,\Omega$

$$I_L = \frac{-7.73 \angle 72.6}{16.77 - j2}$$

$$= \frac{-7.73 \angle 72.6}{16.88 \angle -6.8} =$$

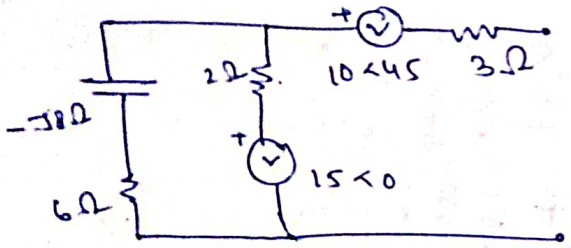
$$= -0.45 \angle 79.4$$

$$= 0.45 \angle -100.6 \text{ A} \quad (\text{Ans})$$

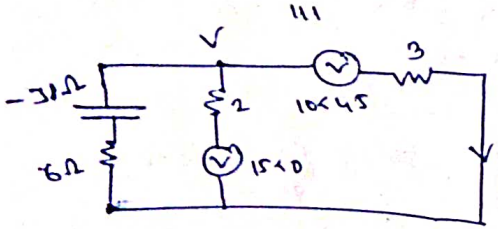
* practical application → loudspeaker

Q2) Obtain Norton's eqv circuit at terminal ~~at given pt~~ A, B.

→



111



$$\frac{V}{6-j8} + \frac{V-15\angle 0}{2} + \frac{V-10\angle 45}{3} = 0$$

$$V\left(\frac{1}{6-j8} + \frac{1}{2} + \frac{1}{3}\right) - \frac{15\angle 0}{2} - \frac{10\angle 45}{3} = 0$$

$$\Rightarrow V\left(\frac{6 + 5(6-j8)}{6(6-j8)}\right) - 7.5 - 3.33\angle 45 = 0$$

$$\Rightarrow V\left(\frac{36 - j40}{36 - j48}\right) - 10.1264\angle 13.4187 - 10.83\angle 45 = 0$$

$$\Rightarrow V = \frac{10.83\angle 45}{\left(\frac{36-j40}{36-j48}\right)}$$

$$V = \frac{10.1264\angle 13.4187}{0.896\angle 5.12}$$

$$= 11.30\angle 8.29 =$$

$$\approx 10.91\angle 8.78$$

$$I_N = 2.18\angle -55.5$$

$$= \frac{10.83\angle 45}{(53.81\angle -48.01)}$$

$$= \frac{10.83\angle 45}{(60\angle -53.13)}$$

$$= \frac{10.83\angle 45}{0.896\angle 5.12}$$

$$= 12.08\angle 39.88$$

$$I_N = \frac{V - 10\angle 45}{3}$$

$$= \frac{10.91\angle 8.78 - 10\angle 45}{3}$$

$$= \frac{10.78 + j1.66 - 7.07 - j7.07}{3}$$

$$= \frac{3.71 - j5.41}{3}$$

$$\frac{53.81\angle -48}{60\angle -53.13} = 0.896\angle 5.13$$

$$\frac{6.55\angle -55.5}{3} = 2.18\angle -55.5$$

$$Z_{th} = \frac{(6-j8)\times 2}{8-j8} + 3$$

$$= \frac{12-16j}{8-j8} + 3$$

$$= \frac{20\angle -53.13}{11.31\angle -45} + 3$$

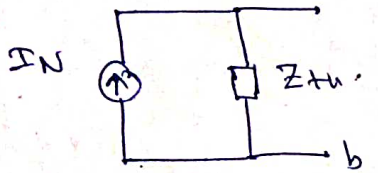
$$= 1.76\angle -8.13 + 3$$

$$= 1.74 + j0.24 + 3$$

$$= 4.74 + j0.24$$

$$= 4.74\angle -2.9\ \Omega$$

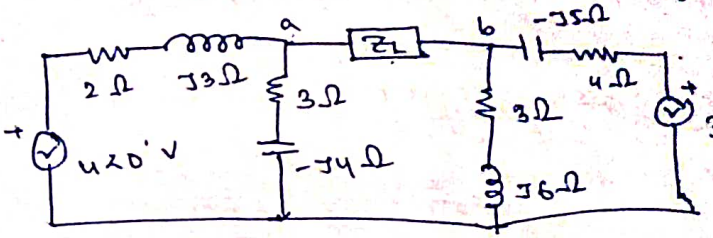
$V = 10.91\angle 8.78$ → try at home!



Norton's equivalent circuit.

Q3) What should be the value of Z_L connected across the terminal ab so that it will draw max power? what is the find amount of max power)

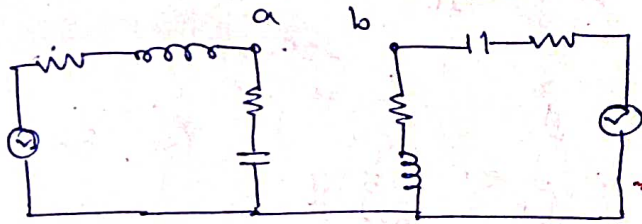
⇒



condition for MPTT → $Z_L = Z_{th}^*$

$$P = \frac{V_{th}}{4R_L}$$

⇒



$6.04 + j0.4227$

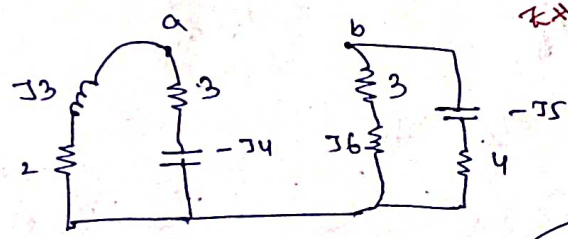
$3.41 + j0.882$

$Z_{th} = 3.53\angle 14.47 + 6.06\angle 4$

$= 9.45 + j1.30$

$= 9.53\angle 7.89°$

⇒



$$Z_{th} = \frac{(2+j3)(3-j4)}{(5-j)} + \frac{(3+j6)(4-j5)}{(7+j)}$$

$(6.70\angle 63.43)$ $(6.40\angle -51.3)$

$42.88\angle 12.13$ $7.07\angle 8.13$

$$= \frac{(3.60\angle 56.30)(5\angle -53.13)}{(5.09\angle -11.30)} = \frac{18.00\angle 3.17}{5.09\angle -11.36}$$

$= 7.07\angle 8.13$

$= 6.06\angle 4$

$= 3.53\angle 14.47$

$Z_{th}^* = 9.53 \angle 7.83°$

$= 9.47 + J1.3$

Z_L will be conjugat of Z_{th}

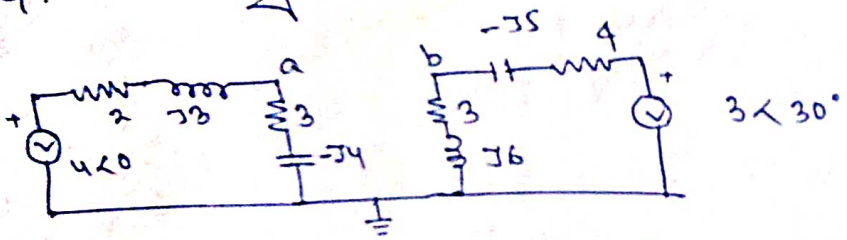
$\boxed{Z_L = Z_{th} \text{ in } \angle}$

In AC if $Z_{th}^* = x + jy$

$Z_L = x - yj$

so, $Z_L = (9.47 - J1.3)\ \Omega$

Next step: finding V_{th}



$V_{th} = V_a - V_b$

$\frac{V_a - 4}{2+J3} + \frac{V_a}{3-J4} = 0$

$\Rightarrow \frac{V_a}{2+J3} - \frac{4}{2+J3} + \frac{V_a}{3-J4} = 0$

$\frac{V_b - 3\angle 30}{4-J5} + \frac{V_b}{3+J6} = 0$

Finding out V_a & V_b.

$\frac{V_a}{2+J3} - \frac{4}{2+J3} + \frac{V_a}{3-J4} = 0$

$V_a\left(\frac{1}{2+J3} + \frac{1}{3-J4}\right) = \frac{4}{2+J3}$

$\Rightarrow V_a\left(\frac{1\angle 0}{3.61\angle 56.31} + \frac{1\angle 0}{5\angle -53.13}\right) = \frac{4\angle 0}{3.61\angle 56.31}$

$\Rightarrow V_a\left(0.277\angle -56.31 + 0.2\angle 53.13\right) = 1.11\angle -56.31$

$\Rightarrow V_a\left(0.154 - J0.23 + 0.12 + J0.16\right) = 1.11\angle -56.31$

$\Rightarrow V_a = \frac{1.11\angle -56.31}{0.274 - J0.07}$

$= \frac{1.11\angle -56.31}{0.283\angle -14.33}$

$= 3.922\angle -41.98$

$\Rightarrow \frac{V_b}{4-J5} - \frac{3\angle 30}{4-J5} + \frac{V_b}{3+J6} = 0$

$\Rightarrow V_b\left(\frac{1}{4-J5} + \frac{1}{3+J6}\right) = \frac{3\angle 30}{4-J5}$

$\Rightarrow V_b\left(\frac{1}{6.4\angle -51.34} + \frac{1}{6.71\angle 63.43}\right) = \frac{3\angle 30}{6.40\angle -51.34}$

$\Rightarrow V_b\left(0.156\angle 51.34 + 0.149\angle -63.43\right) = 0.468\angle 81.34$

$\Rightarrow V_b\left(0.0974 + J0.122 + 0.067 - J0.133\right) = 0.468\angle 81.34$

$\Rightarrow V_b = \frac{0.468\angle 81.34}{0.1644 - J0.011} = \frac{0.468\angle 81.34}{0.165\angle -3.83}$

$= 2.836\angle 85.17$

$V_{th} = V_{ab} = V_a - V_b$

$= 3.92 \angle -41.83 - 2.84 \angle 85.3$

$= 2.92 - j2.61 - (0.23 + j2.83)$

$= 2.69 - j5.44$

$= 6.06 \angle -63.69$

Now, $P_{L_{max}} = \dfrac{|V_{th}|^2}{4R_{th}}$

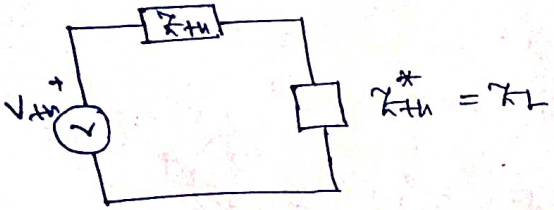
$= \dfrac{36.72 \angle -127.38}{4 \times (9.47)}$

as we have found

$Z_{th} = 9.47 + j1.3$ (R_{th}, X_{th})

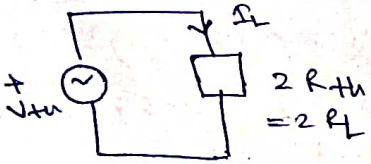
$P_{L_{max}} = \dfrac{36.72 \angle -127.38}{37.88}$

$= 0.969 \angle -127.38$ watt.



} imaginary part will be vanished

$\Rightarrow$

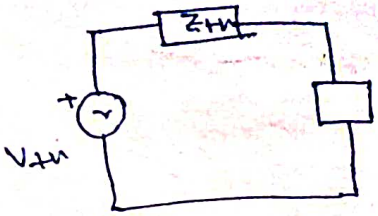


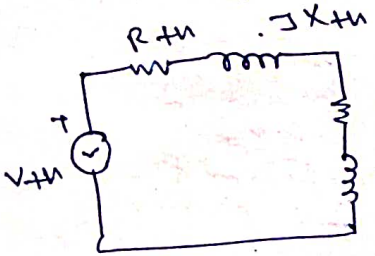
Pure resistive ckt

$Z = R + X$

↓ Real part ↓ imaginary

Pure resistive ckt power $P = I_L^2 R_L$







$I_L = \dfrac{V_{th}}{2R_{th}}$

$= 0.31 \angle -63.69$ ✓

$= \left(\dfrac{V_{th}}{2R_{th}}\right)^2 \times 2R_{th}$

$= \dfrac{36.72}{2R_{th}}$

$P = (0.31)^2 \angle -127.38 \times 2 \times 9.47$

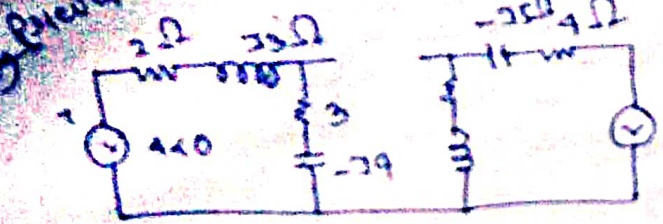
$= 1.82 \angle -127.38$

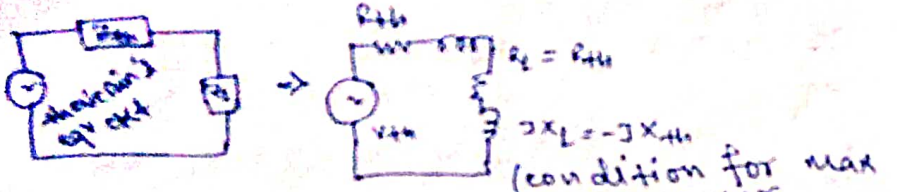
$I_L = \dfrac{V_{th}}{2R_{th}}$

$= \dfrac{6.06 \angle -63.69}{9.47 \times 2}$

$= 0.31 \angle -63.69$

circuit theory

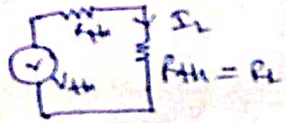




(condition for max power transfer)

$$I_L = \frac{V_{th}}{2R_{th}}$$

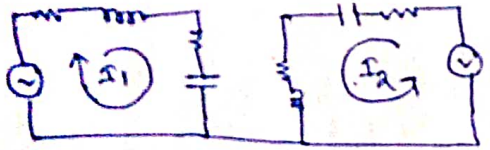
$$P_{max} = I_L^2 R_L$$



$$= \frac{V_{th}^2}{4R_{th}^2} \times R_L = \frac{V_{th}^2}{4R_{th}}$$

(solved previously)

[For MPTT this above part will always be remain same]



$$I_1 = \frac{4}{2+j3+3-j4} = \frac{4}{5-j1} = \frac{4}{5.1\angle -11.31}$$

$$= 0.784 \angle 11.31 \text{ A}$$

$$I_2 = \frac{3\angle 30}{4-j5+3+j6} = \frac{3\angle 30}{7+j1} = \frac{3\angle 30}{7.07\angle 8.13}$$

$$= 0.424 \angle 28.87 \text{ A}$$

$$V_a = I_1(\quad) \qquad V_b = I_2(3+j4)$$

$$= (0.784\angle 11.3)(5\angle -53.13) \qquad = (0.424\angle 21.87)(5\angle 53.13)$$

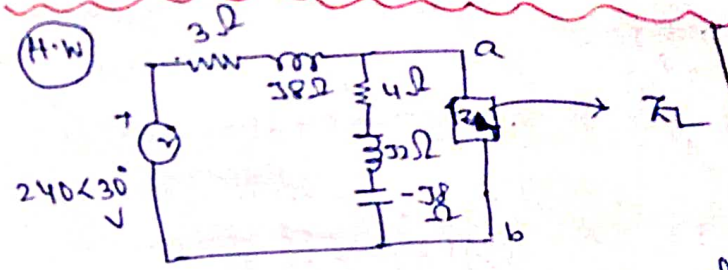
$$= 3.92 \angle -41.8 \text{ V} \qquad = 2.84 \angle 85.3 \text{ V}$$

$$V_{th} = V_a - V_b = (3.92\angle -41.8) - (2.84\angle 85.3)$$

$$= 2.69 - j5.44$$

$$= 6.06 \angle -63.69 \text{ V}$$

(H.W)

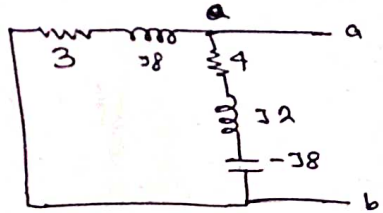


what should be the value of Z_L connected across the terminal ab so that it will draw max power?
find amount of max power?

$$Z_L = Z_{th}^*$$

$$P = \frac{V_{th}^2}{4R_L}$$

⇒ finding Z_{th}



$$Z_{th} = (3+j8) \,||\, (4-j6)$$

$$= \frac{(3+j8)(4-j6)}{7+j2}$$

$$= \frac{(8.544\angle 69.44)(7.21\angle -56.31)}{(7.28\angle 15.95)}$$

$$\Rightarrow Z_{th} = \frac{61.60\angle 13.13}{7.28\angle 15.95}$$

$$= 8.461 \angle -2.82^\circ = 8.451 - j0.42$$

$$Z_L = Z_{th}^* = 8.451 + j0.42$$

Laplace Transformation :-

↓

eqn with integration & diff- that is complex ckt can be easily solved using this :-

Q1) consider a f/n $f(t)$ which is to be continious & defined for values of $t \geq 0$, the laplace transform is then given by

$$\mathcal{L}[f(t)] = F(s)$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-st}\, u(t)\, dt$$

$$= \int_{0}^{\infty} f(t)\, e^{-st}\, dt$$

⇒ where

$s = \sigma + j\omega$ — the complex frequency variable

The Inverse laplace Transformation is defined by

$$f(t) = \mathcal{L}^{-1}[f(s)] = \frac{1}{2\pi j} \int_{\sigma_0 - j\omega}^{\sigma_0 + j\omega} f(s)\, e^{st}\, ds$$

where σ_0 is a real positive constant.

* due to the property of uniqueness of laplace transform it is not essential to perform the above complex integration to obtain $f(t)$

the table of transform pair may conviniently be used.

property of uniqueness :- If laplace transform of two time f/n $f_1(t)$ & $f_2(t)$ are the same f/n $F(s)$ in the 's' domain then $f_1(t) = f_2(t)$

in other words, there can't be more than one f/n having the same laplace transform $F(s)$.

* laplace transform of some typical f/n :-

1) $\mathcal{L}[u(t)] = \int_0^{\infty} u(t)\, e^{-st}\, dt$

$$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = 0 + \frac{1}{s} = \frac{1}{s}$$

2) $\mathcal{L}[e^{-at}] = \int_0^{\infty} e^{-at} e^{-st}\, dt$

$$= \int_0^{\infty} e^{-(s+a)t}\, dt = \frac{1}{s+a}$$

3) Now, $f(t) = \sin \omega t$

Euler's form

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

$$e^{j\omega t} + e^{-j\omega t} = 2\cos\omega t$$

$$\Rightarrow \cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin\omega t = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

Now, $\mathcal{L}[\sin\omega t] = \int_0^\infty e^{-st}\frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)dt$

$$= \frac{1}{2j}\int_0^\infty \left\{e^{-(s-j\omega)t} - e^{-(s+j\omega)t}\right\}dt$$

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

as we solved previously

$$\int_0^\infty e^{-(s+a)t}dt = \frac{1}{s+a}$$

Now,

$$= \frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$= \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}[\cos\omega t] = \frac{s}{s^2+\omega^2}$$ [solve in home]

* Laplace Transform pairs :-

$f(t)$	$F(s)$
$Kf(t)$	$KF(s)$
$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\frac{d}{dt}f(t)$	$sF(s) - f(0)$
$\int_0^t f(t)$	$\frac{F(s)}{s}$
$e^{-at}f(t)$	$F(s+a)$ ✓
$\delta(t)$	1
$u(t)$	$1/s$
t	$1/s^2$
* e^{-at}	$\frac{1}{(s+a)}$
$\sin\omega t$	$\omega/s^2+\omega^2$
$\cos\omega t$	$s/s^2+\omega^2$
te^{-at}	$\frac{1}{(s+a)^2}$
K	$\frac{K}{s}$
$Ku(t)$	$\frac{K}{s}$

Initial value theorem

If the laplace transforms $F(s)$ & $F(p)$ respectively of a fn $f(t)$ & its derivative $f'(t)$ exists then

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Proof:- From Laplace Tr. pair table

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

$$\int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = sF(s) - f(0)$$

Apply $\lim_{s\to\infty}$ on both sides

$$\lim_{s\to\infty} \int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = \lim_{s\to\infty} [sF(s) - f(0)]$$

$$\Rightarrow \quad 0 = \lim_{s\to\infty} sF(s) - f(0)$$

$$\text{ie} \quad f(0) = \lim_{s\to\infty} sF(s)$$

$$\boxed{\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)}$$

PROVED.

Final value theorem:-

$$\boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

$$\int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = sF(s) - f(0)$$

Apply $\lim_{s\to 0}$ on both sides

$$\lim_{s\to 0} \int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = \lim_{s\to 0} [sF(s) - f(0)]$$

$$\Rightarrow \int_0^\infty \frac{d}{dt} f(t)\, dt = \lim_{s\to 0} [sF(s) - f(0)]$$

$$\Rightarrow [f(t)]_0^\infty = \lim_{s\to 0} sF(s) - f(0)$$

$$\Rightarrow f(\infty) - f(0) = \lim_{s\to 0} sF(s) - f(0)$$

$$\Rightarrow \boxed{\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)}$$

Q1) Find $f(t)$ if $F(s) = \dfrac{6(s+2)}{(s+1)(s+3)(s+4)}$

↳ use case of Partial fraction expansion

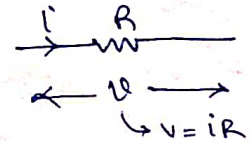
$$A = xF(s)\Big|_{x=0}$$

9/9/24 Circuit theory & Networks-14

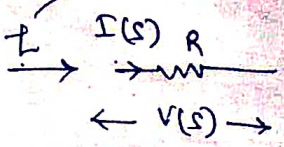
R, L, C in S-domain :-

applying Laplace on both sides

R

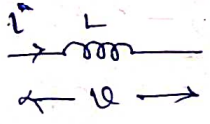


$\xrightarrow{L}$



$V(s) = R\,I(s)$

L



$\Rightarrow v = L\frac{di}{dt} \xrightarrow{L} V(s) = L[sI(s) - i(0)]$

Impedance form

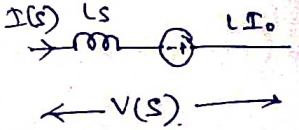
$\Rightarrow V(s) = LsI(s) - LI_0$

or $I(s) = \frac{V(s)}{Ls} + \frac{I_0}{s}$ where $I_0 = i(0)$

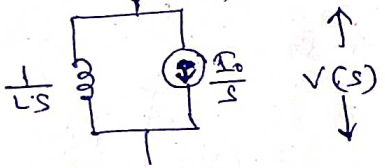
Admittance form

Equivalent circuit.

Impedance form



Admittance form



Admittance form → V V V

Admittance → Reciprocal of Impedance

$Y = \frac{1}{Z}$

$$\frac{6(s+2)}{(s+1)(s+3)(s+4)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = (s+1)F(s)\Big|_{s=-1} = \frac{6(s+2)}{(s+3)(s+4)}\Big|_{s=-1} = 1$$

$$B = \frac{6(s+2)}{(s+1)(s+4)}\Big|_{s=-3} = 3$$

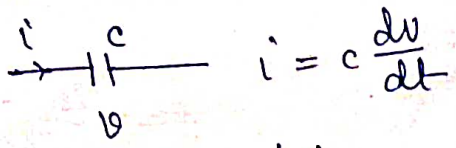
$$C = \frac{6(s+2)}{(s+1)(s+3)}\Big|_{s=-4} = -4$$

$$F(s) = \frac{1}{s+1} + \frac{3}{s+3} - \frac{4}{s+4}$$

taking inverse laplace

$$f(t) = e^{-t} + 3e^{-3t} - 4e^{-4t}$$

capacitor



$$i = C\frac{dv}{dt}$$

$\mathcal{L}$

$$I(s) = C[sV(s) - V(0)]$$

$$\Rightarrow \boxed{I(s) = (Cs)V(s) - CV_0}$$

where $V_0 = v(0)$

KCL form or Admittance form

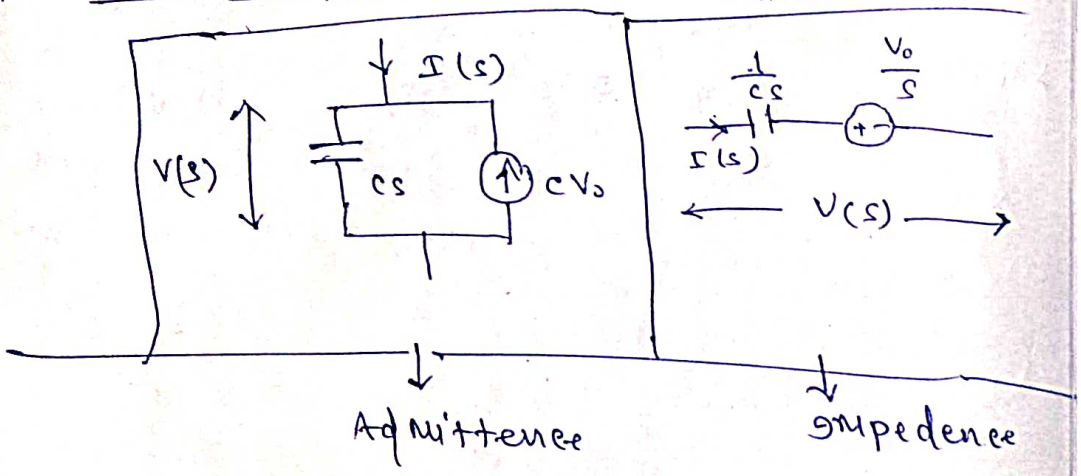
admittance (Y) as current = voltage × admittance

$$\Rightarrow \boxed{V(s) = \frac{I(s)}{Cs} + \frac{V_0}{s}}$$

→ KVL form (Impedence form)

$\frac{1}{Cs}$ (Impedence) as voltage = $\frac{\text{current}}{\text{impedence}}$

$\Rightarrow$ circuit diagram:-

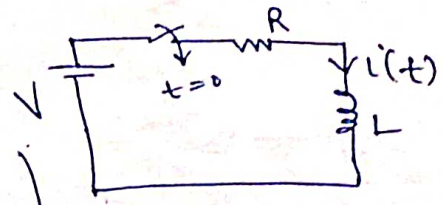


that (RL) & (RC)

last year → RC (derivation)

$\Rightarrow$ consider a series RL circuit as shown. the switch is closed at $t=0$.

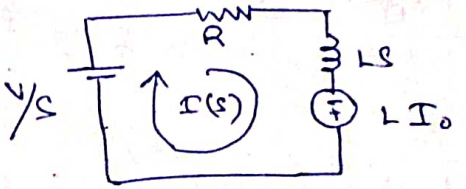
Find the current $i(t)$ by laplace transform-



this V is constant. Sol. in previous it was sin.

Transforming the given ckt into laplace domain

$\mathcal{L}$



in KVL form

$$\frac{V}{S} = RI(s) + LsI(s) - LI_0$$

$$I(s)[R + Ls] = \frac{V}{S} + LI_0$$

$$I(s) = \frac{V/S}{R+Ls} + \frac{LI_0}{R+Ls}$$

$$= \frac{V/S}{L(s + R/L)} + \frac{LI_0}{L(s+R/L)}$$

$$= \frac{V/L}{s(s + R/L)} + \frac{I_0}{s + R/L}$$

$$\frac{V/L}{s(s+R/L)} = \frac{A}{s} + \frac{B}{s+R/L}$$

$$A = \left.\frac{s(V/L)}{s(s+R/L)}\right|_{s=0} = \frac{V}{R}$$

$$B = \left.\frac{V/L}{s}\right|_{s=-R/L} = -\frac{V}{R}$$

$$I(s) = \frac{V/R}{s} - \frac{V/R}{s+R/L} + \frac{I_0}{s+R/L}$$

$$\mathcal{L}^{-1}\{i(t)\} = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} + I_0 e^{-\frac{R}{L}t}$$

$1 \to \frac{1}{s}$, $s \to 1$

$$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right) + I_0 e^{-t/\tau}$$

where $\tau = \frac{L}{R}$ = time const.

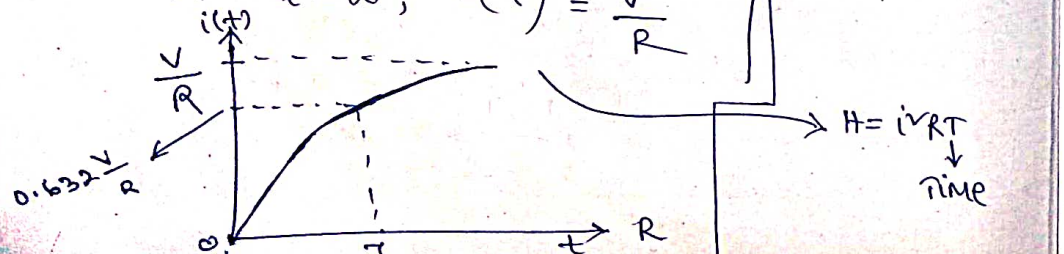
* if initial condition is zero

$$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

* at $t=0$, $i(t) = 0$

at $t=\infty$, $i(t) = \frac{V}{R}$

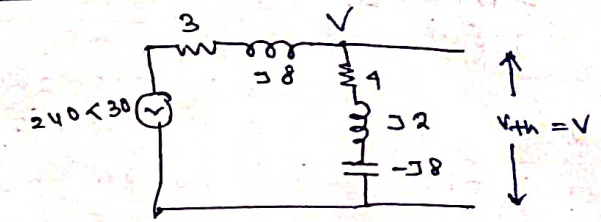
at $t = \tau$, $i(t) = 0.632\frac{V}{R}$

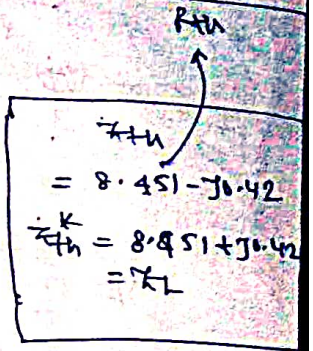


$H = i^2RT$ → time

Time const :- at that value of time at which the current reaches 63.2% of its final value that is $\frac{V}{R}$

(H.W) solution :-





$$\Rightarrow \frac{V - 240\angle 30}{3+j8} + \frac{V}{4-j6} = 0$$

$$\Rightarrow V\left(\frac{1}{3+j8} + \frac{1}{4-j6}\right) = \frac{240\angle 30}{3+j8}$$

$$\Rightarrow V\left(\frac{1\angle 0^\circ}{(8.544\angle 69.44)} + \frac{1\angle 0^\circ}{(7.21\angle -56.31)}\right) = \left(\frac{240\angle 30}{8.544\angle 69.44}\right)$$

$$\Rightarrow V\left((0.117\angle -69.44) + (0.138\angle 56.31)\right) = (28.09\angle -39.44)$$

$$\Rightarrow V(0.041 - j0.11 + 0.077 + j0.115) = 28.09\angle -39.44$$

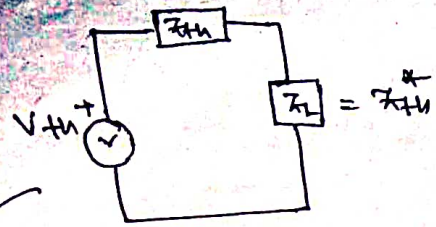
$$\Rightarrow V(0.118 + j0.005) = 28.09\angle -39.44$$

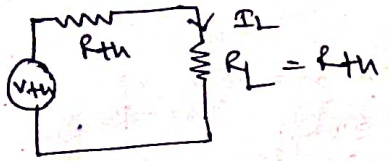
$$\Rightarrow V = \frac{28.09\angle -39.44}{(0.118\angle 2.43)} = 246.52\angle -41.87$$

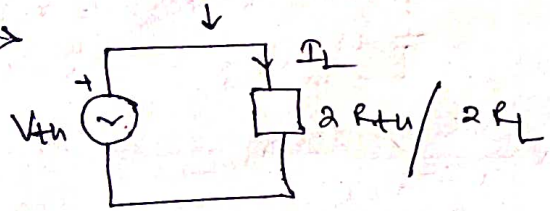
$$(P_L)_{max} = \frac{|V_{th}|^2}{4R_{th}} = \frac{(246.52)^2}{4 \times 8.451} = 1797.78\ W$$

$Z_L = 8.451 + j0.42 = 8.46\angle 2.84$

$(P_L)_{max} = 1797.78$ watt





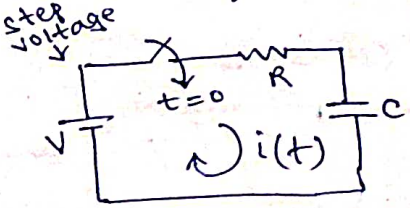


$$I_L = \frac{V_{th}}{2R_{th}} = \frac{V_{th}}{2R_L} = \frac{216.52\angle -41.87}{2 \times 8.951} = 14.59\angle -41.87$$

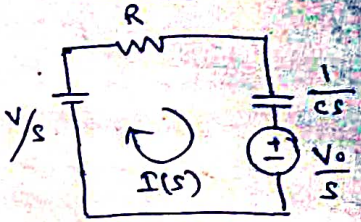
pure resistive ckt.

Find $i(t)$ at $t = 0^+$ using laplace transform?

RC circuit



$\xrightarrow{\mathcal{L}}$



$$I(s) = \left(\frac{V - V_0}{s}\right) \Big/ \left(R + \frac{1}{Cs}\right)$$

$$= \frac{(V - V_0)/R}{s + \frac{1}{RC}} = I(s)$$

$\xrightarrow{\mathcal{L}^{-1}}$

$$\boxed{i(t) = \frac{V - V_0}{R} e^{-\frac{t}{RC}}}$$

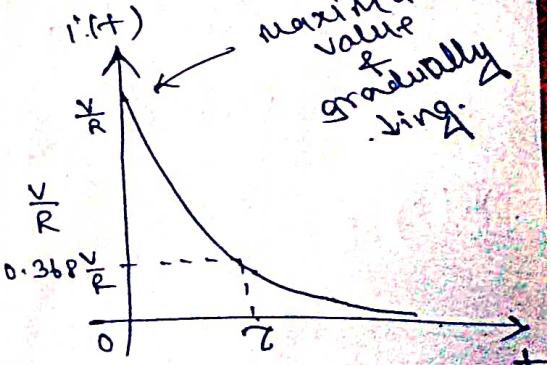
or $\quad i(t) = \left(\frac{V - V_0}{R}\right) e^{-\frac{t}{\tau}}$ where $\tau = RC$ = Time const

ignore previous charge that is V_0

at $t = 0$, $\quad i(t) = \frac{V}{R}$

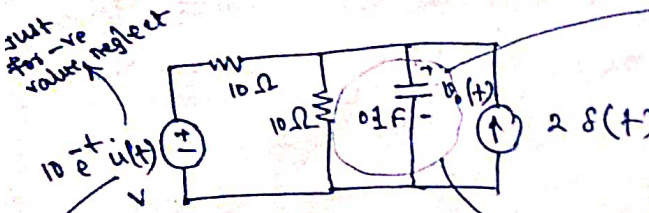
at $t = \infty \quad i(t) = 0$

at $t = \tau$, $i(t) = 0.368 \frac{V}{R}$

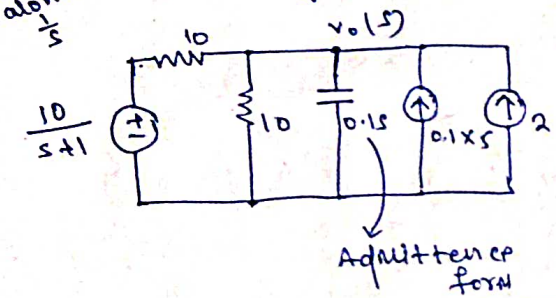


Hence RC Time constant is that value of time at which the current becomes 36.8% of its initial value.

Q1) find $v_o(t)$. Assume $v_o(0) = 5\,V$



writing $\frac{1}{cs}$ or cs



you can't transform Admittance form as you are applying KCL we are doing $\frac{V}{\text{impedence}}$

so VXCS

$$\frac{V_o(s) - \frac{10}{s+1}}{10} + \frac{V_o(s)}{10} + V_o(s)(0.1s) - 0.5 - 2 = 0 \quad \text{(i)}$$

$$\Rightarrow V_o(s)\left\{\frac{1}{10} + \frac{1}{10} + 0.1s\right\} = \frac{10}{s+1} + 2.5$$

$$\Rightarrow V_o(s)\left\{\frac{1}{5} + 0.1s\right\} = \frac{1}{s+1} + 2.5$$

$$V_o(s) = \frac{\left(\frac{1}{s+1} + 2.5\right)}{\left(\frac{1}{5} + 0.1s\right)}$$

$$= \frac{(1 + 2.5s + 2.5)\,5}{(1 + 0.5s)(s+1)}$$

$$= \frac{5s + 12.5s + 12.5}{s + 0.5s^2 + 1 + 0.5s}$$

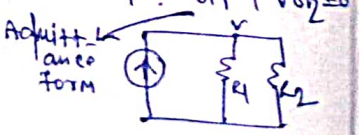
$$= \frac{17.5 + 12.5s}{1.5s + 0.5s^2 + 1}$$

(wrong method)

$-I + \frac{V}{R_1} + \frac{V}{R_2} = 0$

or

$-I + V G_1 + V G_2 = 0$



$I = \frac{\text{voltage}}{\text{impedence}}$

= voltage x admittence

$I = V \times cs \rightarrow$ Admittance

$= \frac{V}{\left(\frac{1}{cs}\right)} \rightarrow$ impedence

From (1) or

$$V_o(s) = \frac{25}{s+2} + \frac{10}{(s+1)(s+2)}$$

$$= \frac{25}{s+2} - \frac{10}{s+2} + \frac{10}{s+1}$$

$$= 25e^{-2t} + 10e^{-t} - 10e^{-2t}$$

$$= 15e^{-2t} + 10e^{-t}$$

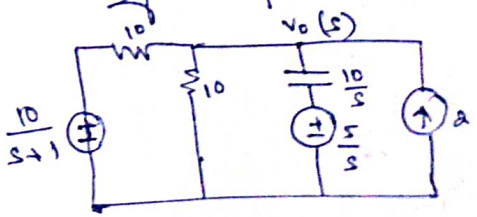
using Partial fraction

$$\frac{10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{10}{s+2}\Big|_{s=-1} = 10$$

$$B = \frac{10}{s+1}\Big|_{s=-2} = -10$$

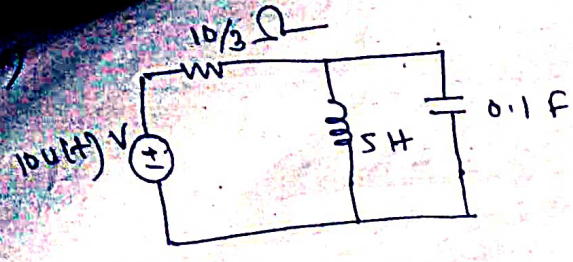
using impedence Method :-



$$\frac{V(s) - \frac{10}{s+1}}{10} + \frac{V(s)}{10} + \frac{V(s) - \frac{5}{s}}{10} - 2 = 0$$

$$\Rightarrow \frac{V(s)}{10} - \frac{1}{s+1} + \frac{V(s)}{10} + V(s)\frac{s}{10} - \frac{1}{2} - 2 = 0$$

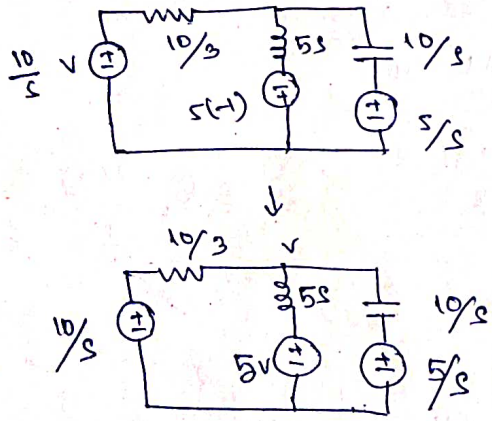
$$\Rightarrow V_o(s) = 15e^{-2t} + 10e^{-t}.$$



Find the value of the voltage across the capacitor assuming that at $t=0$ -1 A flows through the inductor & +5 V is across the capacitor -

$$\Rightarrow \boxed{\begin{aligned} i_L(0) &= -1\ A \\ v_c(0) &= 5V \end{aligned}}$$

Transforming into Laplace domain:



$$\Rightarrow \frac{V-\frac{10}{s}}{\frac{10}{3}} + \frac{V-5}{5s} + \frac{V-\frac{5}{s}}{\frac{10}{s}} = 0$$

$$\Rightarrow \frac{V}{\frac{10}{3}} - \frac{\frac{10}{s}}{\frac{10}{3}} + \frac{V}{5s} - \frac{5}{5s} + \frac{V}{\frac{10}{s}} - \frac{\frac{5}{s}}{\frac{10}{s}} = 0$$

$$\Rightarrow V\left(\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right) - \frac{3}{s} - \frac{1}{s} - \frac{1}{2} = 0$$

$$\Rightarrow V = \frac{\left(\frac{3}{s} + \frac{1}{s} + \frac{1}{2}\right)}{\left(\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right)}$$

$$= \frac{3s + 10 + 5s}{3s + 2 + s^2} = \frac{5s+40}{(s+1)(s+2)}$$

$$= \frac{8s+10}{s^2+2s+s+2} = \frac{8s+10}{(s+1)(s+2)}$$

Partial fraction:

$$\frac{5s+40}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

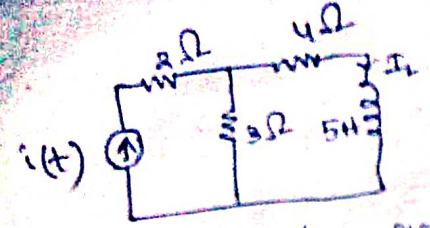
$$A = \left.\frac{5s+40}{s+2}\right|_{s=-1} = 35$$

$$B = \left.\frac{5s+40}{s+1}\right|_{s=-2} = -30$$

$$V = \frac{35}{s+1} - \frac{30}{s+2}$$

$$= 35e^{-t} - 30e^{-2t}$$

Q)



1) unit step response (i(t))
2) Impulse response (h)

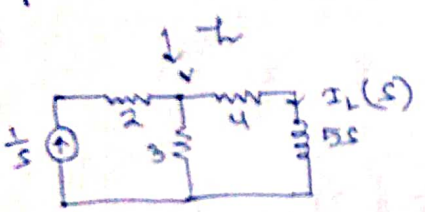
Find the unit step response for I_L.

Given $I_L(0^-) = 0$

0^- → before switching
0^+ → after switching

⇒ As unit step response
so $i(t) = u(t)$

Transforming the ckt into laplace domain



when there is a current source, then any preference b/w the 'V' of current source will be ignored

KCL

$$\frac{V(s)}{3} + \frac{V(s)}{4+5s} - \frac{1}{s} = 0$$

$$\Rightarrow V(s)\left(\frac{1}{3} + \frac{1}{4+5s}\right) - \frac{1}{s} = 0$$

$$\Rightarrow V(s) = u(t) + \frac{1}{4}\left(\frac{1}{\frac{1}{4}+\frac{s}{4}s}\right)$$

$$= u(t) + \left(\frac{1}{4}\right) e^{-\frac{s}{4}t}$$

$$V = \frac{u(t)}{\frac{1}{3} + \left(\frac{1}{4}e^{-\frac{s}{4}t}\right)}$$

as we need to find $I_L(s)$

$$V(s)\left\{\frac{1}{3} + \frac{1}{4+5s}\right\} - \frac{1}{s} = 0$$

$$\Rightarrow V(s)\left[\frac{4+5s+3}{3(4+5s)}\right] = \frac{1}{3}$$

$$\Rightarrow V(s) = \frac{3(4+5s)}{(7+5s)3}$$

NOW, $I_L(s) = \frac{V(s)}{4+5s}$

$$= \frac{3}{s(5s+7)}$$

$$= \frac{3/5}{s(s+7/5)} = \frac{A}{s} + \frac{B}{s+7/5}$$

⇒ when $s=0$, $A = \frac{3/5}{7/5} = \frac{3}{7}$

when $s = -7/5$, $B = \frac{3/5}{-7/5} = -\frac{3}{7}$

$$I_L(s) = \frac{3/7}{s} + \frac{(-3/7)}{s+7/5}$$

$$= \frac{3}{7}u(t) - \frac{3}{7}e^{-\frac{7}{5}t}$$

$$= \frac{3}{7}\left(u(t) - e^{-\frac{7}{5}t}\right)$$

$$= \frac{3}{7}\left(1 - e^{-\frac{7}{5}t}\right)$$

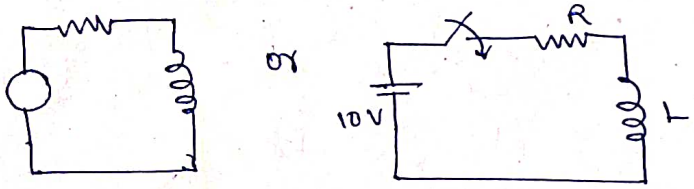
By current division rule.

$$\Rightarrow \quad \Omega_L(s) = \frac{1}{s} \times \frac{3}{3+4+5s}$$

$$= \frac{3/5}{s(s+7/5)}$$

Q2) ~~white~~ coil has resistance of 2.5 Ω & its time const. is 1.6 seconds, determine a) the current 1s after 10 V is applied. b) the time taken for the current to attend (to reach) half of its final value.

* coil: Inductor with Internal resistance



a) $\tau = \frac{L}{R} = 1.6$, $t = 1s$, $V = 10V$

$$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$$

$$= \frac{10}{2.5}\left(1 - e^{-\frac{1}{1.6}}\right)$$

$$= \frac{10}{25} \times 10 \left(1 - e^{-\frac{10}{16}}\right)$$

$$= 4\left(1 - e^{-\frac{5}{8}}\right) = 1.86 \text{ A}$$

b) half of $\left(\frac{V}{R}\right) = \frac{V}{2R} = 2 \text{ A}$

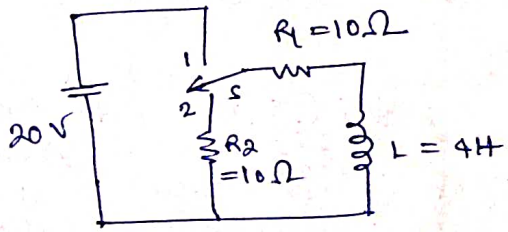
$$\frac{V}{2R} = \frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$1 - e^{-\frac{t}{1.6}} = 0.5 \quad \Rightarrow \quad e^{-\frac{t}{1.6}} = 0.5$$

$$\Rightarrow \quad \frac{1}{e^{\frac{t}{1.6}}} = \frac{1}{2} \quad \Rightarrow \quad e^{\frac{t}{1.6}} = 2 \quad \Rightarrow \quad \frac{t}{1.6} = \ln 2$$

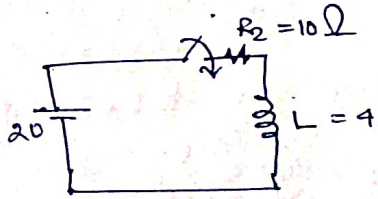
$$\Rightarrow \quad t = 1.6 \times 0.693 = 1.11 \text{ s}.$$

Q3) the ckt in the fig. is initially under steady state condition, the switch is moved from position '1' to position '2' at $t=0$. Find the current after switching.



SPDT switch → single pole double throw

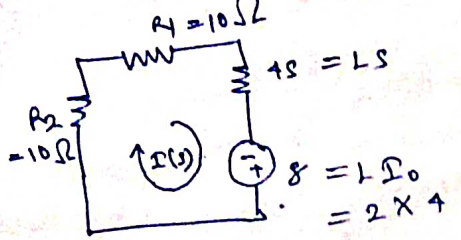
Draw the ckt when switch is in pos-1



$i(0^-) = \frac{V}{R_1} = 2 \text{ A}$

inital value of switching at which 'i' is final valued.

position (2)

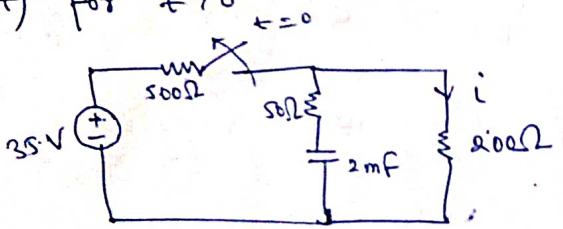


when Inductor is charging & discharging the polarity of voltage is opposite to each other

$$\Rightarrow I(s) = \frac{8}{4s+20} = \frac{2}{s+5}$$

$$i(t) = 2e^{-5t}$$

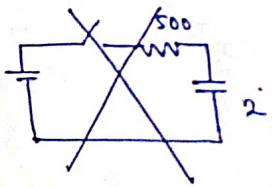
Q4) consider the ckt shown, the switch was in closed position for a long time, it is opened at time $t=0$, find the current $i(t)$ for $t>0$

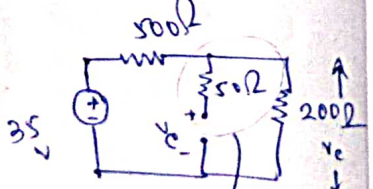


steady state ke ckt ($t=\infty$)

· i at capacitor at $t=\infty$ = 0

ckt before switching





there no voltage drop

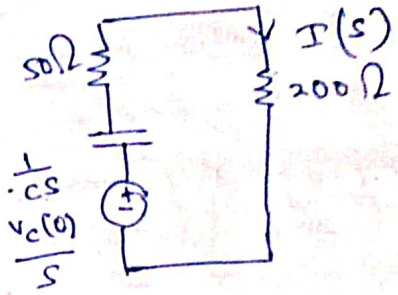
by VDR = $35 \times \frac{200}{700} = 35 \times \frac{200}{500+200} = 10\,V$

$$\cdot \frac{V_c - 35}{500} + \frac{V_c}{200} = 0$$

$$V_c\left[\frac{1}{500} + \frac{1}{200}\right] = \frac{35}{500}$$

$$\Rightarrow V_c\left[\frac{7}{1000}\right] = \frac{35}{500} \Rightarrow V_c = 10V = v_0(s)$$

$$\Rightarrow$$



$$\frac{1}{cs} = \frac{1}{2\times10^{-3}s} = \frac{10^3}{2s} = \frac{500}{s}$$

$$V_0(s) = 10$$

$$\Rightarrow I(s) = \frac{10/s}{250 + \frac{500}{s}} = \frac{10}{250s+500} = \frac{10}{250(s+2)}$$

$$= \frac{1}{25}e^{-2t}.$$

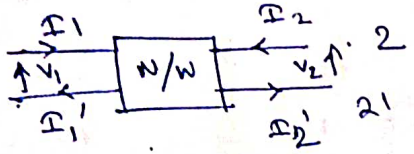
Two port Network :-

A port is a pair of terminals through which the current may enter or leave.

The port is an access to the network.

the port condition is that :

* the current entering the port must be equal to the current leaving the port.

⇒ 

} It's a passive network (as no source is present)

$I_1 = -I_1'$
$I_2 = -I_2'$ } port condition

* To ~~correct~~ characterise a two port network. the terminal quantities V_1 & V_2, I_1 & I_2 are to be related hence at a time, two out of four var. are taken as Independent var. & the other two as dependent var.

$^4C_2$ combinations = 6 combinations are possible

hence there are 6 set of parameters

→ 4 (we will study)

(1.) open circuit parameters;- or Z parameters :- or Impedence Parameters

I) Ind. → I_1 & I_2
Dep. → V_1 & V_2 } step-I (nominating Ind. & Dep var).

II) $[V_1, V_2] = f\{I_1, I_2\}$ } characteristic equation of Z para. (step-II)

⇒ $V_1 = Z_{11} I_1 + Z_{12} I_2$ —(i)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —(ii)

or

in matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ } In matrix form (matrix, vector)

or

$$[V] = [Z][I]$$ } In concise matrix form

(iii) Determination & nomenclature of parameters i.e. Z para.

Impedence ← $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

Input driving pt impedence

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

reverse transfer impedence.

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

forward transfer impedence

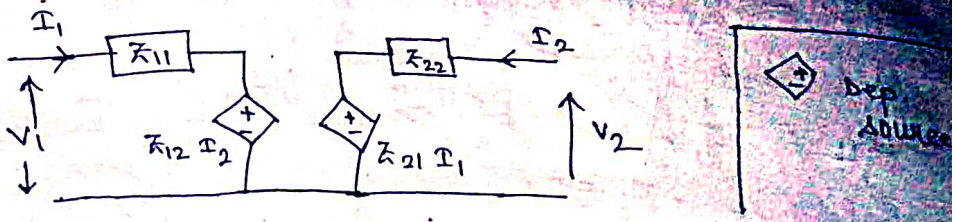
$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

output driving pt impedence

if only input is involved then input para. same for output

input & output
- both → transfer
- only. input → driving or output pt.
  - forward $\left(\frac{out}{in}\right)$
  - reverse $\left(\frac{in}{out}\right)$

(iv) equivalent circuit :-
eqn (i) & (ii) in ckt form



** these parameters are found by $I_1=0$ / $I_2=0$ so open ckt parameter **

(2) ~~short ckt~~ short circuit parameter.

(y-parameters / Admittance)

I) Ind → V_1 & V_2
Dep → I_1 & I_2

II) $[I_1, I_2] = f\{V_1, V_2\}$

$I_1 = y_{11} V_1 + y_{12} V_2$

$I_2 = y_{21} V_1 + y_{22} V_2$

or

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [y][V]$$

II) $[I] = [Y][V]$

$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ → Input driving pt admittance and similarly so on

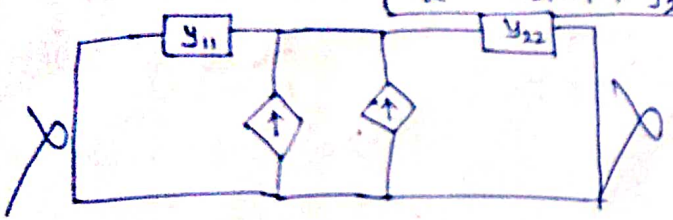
$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$

$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

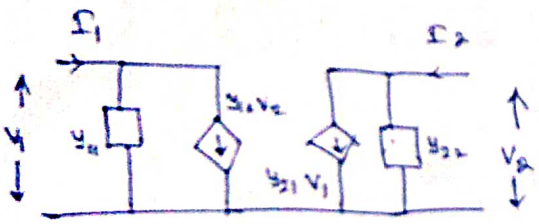
$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

IV) Eqv ckt :-

$I_1 = y_{11}V_1 + y_{12}V_2$
$I_2 = y_{21}V_1 + y_{22}V_2$ → KCL



current should be parallel with admittance



✓

③ Hybrid parameter or h

II) $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

Ind : I_1, V_2

$V_1 = h_{11}I_1 + h_{12}V_2$
$I_2 = h_{21}I_1 + h_{22}V_2$

III) $h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Input driving pt impedance

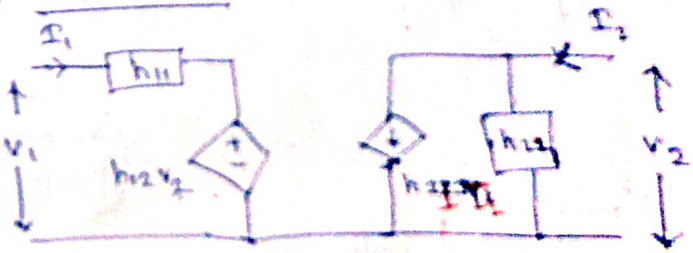
$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ forward current gain

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ reverse voltage gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ output driving pt admittance

q^ve → attenuation

IV) eqv ckt :-



$> h_{21}I_1$

(A) ABCD parameter / Transmission line parameter / chain parameter / T-parameter :-

I) $V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

II) V_2 & I_2 are Indep.

$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

III) $A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ $D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$

A → reverse voltage gain

B → reverse transfer impedence

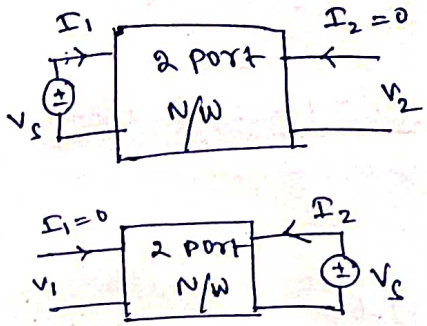
C → reverse transfer Admittance

D → reverse current gain

IV) Not possible

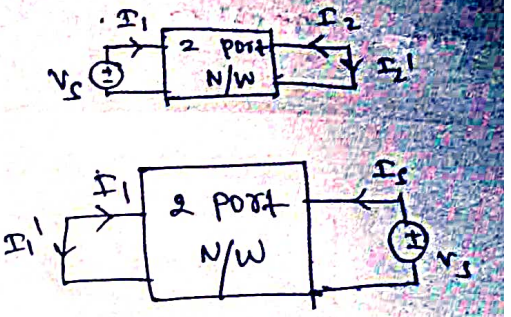
N/W → Network

condition for symmetry



** even in the cond^n of the ports are interchanged, the behaviour of the N/W should remain same i,e. ports can be interchanged without changing the port voltages & current. in other words, if the impedence measured at one port is equal to the impedence measured at other port, then the network will be said to be symmetrical or said to poses the property of symmetry

condition for Reciprocity

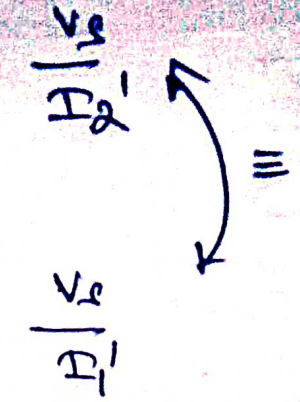


** the ratio of the exitation to the responce should be same even if the pos^{n}s of the exitat^n & responce are interchanged. in other words if the ratio of the voltage at one port to the current at other port is same even if the pos^n of the voltage & current are interchanged, then the N/W will be said to reciprocal or said to poses the property of reciprocity.

$\frac{V_s}{I_1} = \frac{V_s}{I_2}$

one port

another



SORRY
Shohaib bhai